Important mathematical concepts at the dawn of science in Byzantium: the philosopher C. M. Psellos

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Abstract

The Byzantine Empire, an empire that lasted over one thousand years, is acknowledged nowadays as the spiritual source of the European intellectual rebirth of the middle ages. What really distinguishes this great state from any other important country of the time is among others a solid conviction of equality of chances in at least a fundamental component of progress: education. What we try to demonstrate here is the fact that natural sciences such as mathematics were treated in the above sense with the utmost respect by remarkable personalities whose traditional role seemed to be far from the common sense of this scientific branch, but in its true nature was nourished and fostered by the essence of mathematics. Diplomacy and highly exerted church leadership intertwined with mathematical exploration.

Keywords: Diplomacy in Byzantium and Mathematics, Concepts of arithmetic progression and probability, C. M. Psellos

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1 Introduction

Church and science have often found themselves rivals in the historical process. The fundamental question how this universe was born from the days of ancient Ionian philosophy till nowadays has caused a lot of controversy whether it is compatible or not with the generally religious supported theory. Nevertheless in all phases of ecclesiastical history there have been great minds, possessed by the obsession of finding the unknown each one in his or her own discipline maintaining at the same time the conviction that this insatiable thirst for knowledge is in itself a sign of God's plain existence. Byzantium has been characterized (Runciman, S.1969, Zakynthinos, D. 1977) a religion centered state, where the second role placement of the natural sciences is an undeniable fact, mainly because of the influence that these sciences could have exerted over the general belief of God's existence and omnipotence. Notwithstanding the countermeasures employed several times by the Christian church to suppress scientific evolution, the Platonic image of the opposite makes its appearance once more with the presence of illuminated church oriented minds, advocates of Plato, who fundamentally contribute to the pioneering development of strong mathematical ideas, while serving at the same time the highest offices that Church can give them. Among these people there is a prominent place for Constantine Mihail Psellos, whose title rector of the rectors or even better as the Spanish speaking sefaraditas describe him maestro de los

maestros. In this article we are trying to demonstrate how very important mathematical ideas such as the arithmetic progression and probability were conceived by such a great mind based on unedited and worn out by the time manuscripts, which were maintained by low profile proponents of the history of science in the Asiatic suburbs of Bosporus, Istanbul. Moreover we are going to witness an international and interreligion collaboration motivated by pure and strict dedication to science and the passion it entails.

2   Life diagram of Psellos

According to the existing historical notes Psellos was born in 1018 in Istanbul, at the time Constantinople, in a wealthy aristocratic family. It is wrongly believed that in Byzantine times to enjoy a proper and fruit-bearing education (Zakynthinos, D. 1977, Laiou, A. 2007) you needed either to come from a wealthy socially recognized family or be closely associated to the church, so that it could take on the expenses of such an education, the case of Cardinal Besario (Petakos, 2009b). The equality of chances that Byzantium has established for the students of the empire is unique in the middle ages and really emanates from the fundaments of this state at the great Konstantinos era, a Christian state based on a time invariant notion of justice. This tradition marked the Christian population of the late Ottoman Empire till the nineteenth century. Nevertheless Psellos talent and insatiable thirst for knowledge was acknowledged at the early stages of his education and the entailing fame was enough to surpass family's socioeconomic strength and ensure him a state subsidy to continue his studies in the rather neglected city but still bearing a rich philosophic atmosphere of Athens. The rich family background in conjunction with his perfect education earned him quite a name that quickly resulted in his becoming a judge and after that an influential political advisor to row of emperors such as Constantine Monomachos, Michael VI and VII Doukas, Constantine Doukas and Romanos Diogenes. His undeniable rhetoric capability-which in his own words was inextricably connected to an arithmetical thinking, a preconception of the Fibonacci numbers (Petakos, 2009)- made him indispensable to all above mentioned emperors and established him as an ideal model of a diplomatic personality. Following the currents of his time caused a number of shifts in his political loyalty even to his closest collaborators that finally put him aside close to his death, despite his everlasting aspirations of getting even higher. As he personally demonstrates in his manuscripts, politics did absorb a no negligible part of his time that he could have devoted to the study of science and his beloved mathematics. His lifetime work Didaskalia Pantodapi that would be freely translated as Teaching on all subjects, the mathematical thinking of his age becomes more than apparent. Mathematics consists of four basic branches arithmetic, geometry, music and astronomy, whose culmination is according to his Platonic predilection the created harmony. It is worth mentioning that this volume is regarded as the first printed book in Greek in 1532, a part

of which torn out by the time was maintained till today at a private selection in the Asiatic suburb of Cengelkoy, Istanbul (Konstantinidis, I. 1992). His other epic work as presented to us refined and eloquently commented by Karalis (Karalis, V.1993) Chronografia is a historical narration of emperors'lifetimes for a sequence of centuries that surely exceeds the scope of an historical book. It is sort of demonstration of an awkward application of Platonic thinking in the every day political practice, at a time where representatives of the Church were sort of condemning ideas strongly associated to Plato, dividing the Byzantine society into two groups, those who were for the great philosopher and those who were against and later on this division became practically equivalent to the discrimination between those who opposed or not the union with the Catholic Church. This latter work made the great Byzantine history investigator Krumbacher (Krumbacher, K. 1897) say that Psellos as a diplomat surely occupies a position between Bacon and Albert the Great.

3 Mathematical thinking of Psellos- arithmetic progression and signed numbers at their dawn

In the Asiatic suburb of Cengelkoy the time neglected manuscripts of Psellos rest for centuries until the mathematical mind of the Greek-Albanian A. Koco will be attracted to them quite unexpectedly. These manuscripts pass on from generation to generation without having someone to deal with them; a patrimony not suitably exploited till the mathematical propensity of a single heir appears. In the same library collection that alternates between the two suburbs Cengelkoy and Kuzguncuk (Bektas, C. 2003) there is also a part of the famous Ionia work of the empress Eudokia Makrembolitissa, characterized by Byzantine Nikhphoros Grigoras as the second Hypatia (Dzielska, M. 1996). Trying to keep trace of what is written in there-together with exploratory comments of Koco in three languages, Turkish, Albanian and Greek, we seem to be enchanted by the first reference to a sort of arithmetic progression concept centuries before it is even stated in mathematical conversations. His name Psellos means the person who stammers, a physical disadvantage that would have impeded anyone from becoming an orator -much less a professor of oratory- but not Psellos who employs his favorite mathematical thinking as a treatment against this by birth downside. Even some of my audience, students or not find this well attended stammering as a pole of attraction. Whether they tell the truth or not I am sure of one thing: I have tried over a long period to make it obey a mathematical rule, that is after every compound of words, which usually comprises of 2 or 3 words, I make a deliberate pause so that the acoustic difference among what I say to be equalized. Although by some interpreted as a trick to avoid pitfalls or by others as an excuse to think and prepare my arguments further, one is really essential: the struggle I have been exercising all these years have provided me with the desired result; an equidifferent- if the translation is the proper one- stammering that really con-

verts a physical inevitable downside to an ally of speech. From that explanation
of his overcoming personal disabilities and especially by intonating this equality
of differences even between sets of words does it not remind us the fundamental
concept of an arithmetic progression? In fact this concept appears itself applied
in such a realistic situation, in the art of convincing, that everyone has more or
less to apply to attain even the easiest of his/her goals. Not to mention that here
the standards of the AMS seem to be fulfilled in a natural and the same time un-
expected way. To support the extent of our allegation that Psellos was that close
to the formation of the arithmetic progression, we will amplify our arguments by
another example taken by the manuscripts in which he goes one step further by
symbolizing what was already predefined in his mind. Psellos had been appointed
as the Consul of the Philosophers, the rector of the rectors, and enjoyed teaching
before his eager student audience. It seems that recurrence of events seemed to
occupy the minds of those students, following the Eastern Orthodox tradition to
commemorate a saint's death on a specific day of the calendar, which also happens
with the Catholic Church but not with the same intensity. When their name day
will reappear on the same weekday seems to be a question that will arise a lot
of controversy. It is funny and at the same time enticing to see how their mind
works on that problem. Once again I conclude that human intuition would like
to have events occur only after a certain invariant number of periods, as Hypatia
would have liked to state. It is probably their fear for the unknown that supports
the conviction that a certain number of steps, an invariable one, that separates
one phase from the other, should govern the inevitable change. The fact that this
happens in varying circles seems sort of disappointing to them. They would have
preferred a relation like this

$B = A \, \pi\lambda\varepsilon ov \, \Omega$

$\Gamma = B \, \pi\lambda\varepsilon ov \, \Omega$

Here we see the Greek capital letters employed in place of sequential numbers
and the adjective-adverb $\pi\lambda\varepsilon ov$ o v to appear between them. $\Pi\lambda\varepsilon ov$ means more
,it is the Italian piu, P that will be used centuries later by Paccioli and Santa
Cruz as the arithmetic symbol of addition. Moreover while Besario addressed to
his opponent and then best friend Cardinal of Spain Pope Alfons de Borja the
Aragonese (Jaho, I. 1909) there he brought to the Pope's attention the change in
the electorate by using the term mas between numbers. We should not forget that
Besario was also advocate of Plato and was educated in Istanbul, where the spiri-
tual influence of Psellos was maintained till the last day of the Byzantine empire.
This last mathematical word of him mas certainly accrues to his great mathemat-
ical intuition and the influence he exerted a century later to Pacioli in defining the
signs of operations (Cajori, F.1993) through the rich of his manuscripts Marcian
library of Venice. Moreover the final letter of the Greek alphabet plays the role of
the invariant quantity, the difference of an arithmetic progression. Its use is really
magnificent, it is the end of the alphabet so he first exceeds all existing letters and
then he comes to this boundary to define the invariant quantity. We should not

forget here that mathematical use of this letter should not go unheeded since it is considered by some mathematicians the starting inspiration point for the symbol of infinity by Wallis (Scriba, C.J. 1970). At this point it is worth mentioning his impression by talking several times with the empress Eudokia Makrembolitissa The empress is really characterized by a natural science mind. That can be partly attributed to her love for the Great pure one of Alexandria-Hypatia-whose astronomical and at the same time remote world related thinking seems to have conquered the empress's mind. She cannot of course admit this fact in public, but thanks God she confides in me, which makes me feel how trustworthy I am to her. It is a pity that this naturally emanated talent really restricts itself to a mathematical trail of thought and cannot be expanded to the duties that her high political position entails. Let us stay a little more on the above-mentioned comment, putting aside its political interpretation. Firstly the great pure one, which in Greek is translated as KAΘAPH is paraphrased in Latin as KATHARI and consequently the affinity with the name KATHERINE, becomes obvious. An affinity that caused a lot of controversy whether St Catherina and Hypatia (Davis, H.T. 1957) have been the same person in the sense that the former has formed the latter's exemplary counterpart, separated only by the rivalry between the prevail of Christian over paganism thought. The term pure one KAΘAPH according to the distinguished Greek philologist Siamaris (Siamaris, D. 2006) is attributed rarely to women personalities that went far beyond their traditional role that society assigned them in antiquity. It is an honorary title in a man centered society that clearly denotes a woman whose dedication to a noble cause such as religion or science becomes a lifestyle and usually is equivalent to her remaining unmarried. (Dzielska, M.1996). Secondly it verifies the assertion of Gregoras that Makrembolitissa, a weak political leader that let her feelings overshadow the nation's survival did possess a scientific mind whose propensity to astronomy and mathematics was impressing. It is really a great pity that pages left from her famous masterpiece Ionia were left in an empty garden house in the Cavus dere area of Uskudar during the first world war by a Christian family that wanted to evade the undesirable military recruitment of the ottoman state. When the family returned after some years to Istanbul, their property was maintained but the temporary proprietor had already got rid of these time stinking illegible pages as he personally described the loss. Fortunately under the auspices of the Bishop Ierotheo part of it had been thankfully transferred to Albania and therefore saved. It would have been at least enlightening to what extent this contact with Psellos had contributed to the evolution of important mathematical concepts.

4   Preconception of probability on behalf of military interest

Finally we will present the so-called Byzantine school of probability by analyzing the conceptualization that led Psellos, as four centuries later Besario, to inaugurate a really new era where the already establish fraction form would acquire a

totally new dimension. The interplay between religion and science, in our case math, goes on and this time it involves 4 languages and three religions. Our source of inspiration the Bishop of Korce in Albania, Ierotheos Jaho (Glavinas, A. 1984) who becomes in an accelerated way Auxiliary bishop upon finding the chest cross of a deceased Patriarch of Constantinople. Jaho purchases this cross by a Muslim Turkish speaking family, whose existence is made known to him by a sefaradite couple, Spanish-speaking Mediterranean Jews. Along with the cross a set of notes draws Jaho's attention, whose intellectual weight for an Eastern Christian priest was underlined by the sefaradite couple. The collection of the manuscripts contains 5 that belong to Psellos, 5 to Besario and 3 others whose handwriting definitely resembles the previous mentioned authors, but the extent of the aftermath interpolation is wide enough to let someone conclude what the original author would like to say. The historical battle of Macikert that essential-ly denotes the end of |the Byzantine reign in Anatolia is studied in detail in a pure mathematical sense of the word by Psellos not in this case for ensuring the duration of the state that lavished him munificently with the laurels of glory and a lot amenities, but for eliminating the new emperor and his political opponent Romanos Diogenes. The man who although uses him as an advisor does not let himself be easily manipulated because he possesses a strong military oriented per-sonality. Psellos employs data from previous given battles between the Byzantine army and the enemy, letting the numbers plays their decisive and fundamental role. He is writing fractions where the numerator represents the Byzantine army and the denominator -always greater than the numerator- represents either the sum of all forces or some other quantity, which unfortunately is not feasible to in-terpret. By writing down the pressure fraction-the term pressure will be explained later- I do see that what RD (Romanos Diogenes) used to support is a total lie. The denominator seems not to change at all while the above one tends to lose as time evolves. It happened before with the Arabs and now Damascus is beyond our border. I have noticed that the pressure also in that case seemed to decrease itself leading to a complete failure. Once again numbers show me the inevitable defeat and alas to the defeated. The term pressure as an interpretation of the probability fraction underlined by A. Koco seems quite attractive to be studied. When we discussed it with students at the Hellenic Naval Academy, two groups of opinion prevailed. The former coincided with the author's opinion that the word pressure attributed to the fraction was a result of the psychological moment situation, the State's life is at stake and at the same time the political power game is about to change hands. Yet the latter explanation provided by a rather mediocre student did cause us to deliberate it that way. This student asserted that pressure could also be the expression in words of the vertical representation of a fraction; it is the weight of the numerator over the denominator. In the original manuscript the vertical representation is obvious and what remains undecided is whether the separation line between the terms of the fraction exists, and if so it is a curved line not a straight one. If we do accept that math is so closely related to imagination,

we could be persuaded by that explanation. Anyway we cannot be sure what the great philosopher was trying to emphasize by attaching that word next to a pure math term. What we can be sure of is that by bringing it to our every day class we do stimulate and foster studentsínterest by the interconnection of History -in other words the human evolution- with Mathematics teaching (Zapico, I. 2006).

5 A concluding remark for the dedicated instructor

Once more the magnificent capital of two empires, the Byzantine and the Ottoman, forms an unavoidable source of information and inspiration at the same time. The conglomerate of cultures, religions and nationalities that characterized the existence of this city proves itself fruitful enough to set aside any sort of geopolitical differences and aroused disputes under a common goal: the advance of science and specifically mathematics. Constantine Mihail Pselos is a diplomatic genius who according to the the time invariant rules of politics is expected to be involved in a series of manipulation schemes necessary to maintain state's sovereignty in equilibrium with preserving his own fulfillment of personal aspirations, the way that even modern well known diplomats behave themselves and earn a position among the best leaders of all times. The Byzantine society with the exception of highly illuminated minds like Besario, condemn this sort of behavior and gives it the worst vent it can possess, an indifference to the scientific work of the philosopher. The multicultural suburbs of Kuzguncuk (Bektas, C. 2003) and Cengelkoy (Konstantinidis, I. 1992) present an ideal place of an exchange of ideas between the Spanish speaking Jewish inhabitants and the abundant at the time Greek-Albanian community, which does surface the collection of unpublished manuscripts attributed to personalities like Psellos. The introduction of one of these manuscripts will close the present article in an effort to embellish the image of the so slandered Byzantine philosopher by pointing out his teaching oriented nature and the sense of justice that Byzantium and only Byzantium had during the middle ages. It is today generally accepted (Laiou, A 2007) that the cultural shape of the European state has its inauguration point in this era and owes a lot to an empire that the Occident let die without essential help. Always when I teach I make the distinction between the two basic sets of students, those who learn for the joy of learning and those who do it as a means to promote some unfulfilled hidden aspiration of theirs. I do appear to be prejudiced toward the first one, the real servants of science and will ever remain loyal to the conviction that education should be accessible to those who feel the zeal to acquire it wherever they may be, wherever they come from and whatever their social backgrounds are, in other words I try to remain Christian at least in my professor's dimension. His last phrase shows sort of guilty for his political handling but at the same time demonstrates a highly professional attitude while teaching. The same comment will appear much later by the philosopher Maimonides, whose stance on education is undeniably path finding for his age (Tewarson, H.T.1988). It gives

birth to a problematic that even nowadays occupies our deliberation when trying
to improve our role in the everyday classroom practice (Mamona, I. 2008).

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