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Parametric Characterization of Ankle Kinematics for Inverse Dynamics–Based Human–Structure Interaction Models --Manuscript Draft--

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Additional Information:	

Question	Response
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Highlights

- Establishes a parametric link between intrinsic gait parameters and ankle kinematics.
- Employs compact Fourier series to describe kinematics, eliminating the need for subject-specific calibration.
- Integrates realistic intra-subject variability to capture natural cycle-to-cycle gait fluctuations.
- Enables efficient integration into inverse dynamics Human-Structure Interaction simulations.

Parametric Characterization of Ankle Kinematics for Inverse Dynamics–Based Human–Structure Interaction Models

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Abstract

Accurate ankle kinematics are essential for inverse dynamics–based Human–Structure Interaction (HSI) models, as foot motion governs load transmission between pedestrians and civil structures. However, the foot dynamic complex exhibits highly subject- and speed-dependent behavior, making its direct modeling challenging and often requiring dedicated experimental measurements for each individual. This study proposes a parametric characterization of ankle kinematics during walking by explicitly linking intrinsic gait parameters to ankle motion during the stance phase. Experimental gait data were collected from ten subjects with different anthropometric characteristics at multiple walking speeds using a low-cost optoelectronic motion capture system based on smartphone video acquisition and reflective markers. The trajectories of the ankle, heel, and metatarsus markers were processed to identify gait events and compute the foot rollover angle, and an intrinsic-parameter-based numerical model was developed to reproduce subject-dependent ankle kinematics without individual experimental calibration. The foot rollover angle is defined using a subject-specific reference value at midstance derived from foot anthropometry and scaled through a normalized background curve expressed as a Fourier series. Horizontal ankle displacement is described using a dimensionless trajectory reconstructed through scaling functions dependent on walking speed and subject height, while vertical ankle displacement is formulated using an anthropometry-based scaling function governed by leg length. All kinematic components are represented using compact Fourier series, enabling efficient reconstruction of ankle motion. The proposed formulation provides a generalizable and scalable representation of ankle kinematics that significantly reduces experimental requirements and is well suited for integration into inverse dynamics and optimization-based HSI simulations.

Keywords: Ankle Kinematics, Human–Structure Interaction, Inverse Dynamics, Parametric Characterization, Intrinsic Gait Parameters

1. Introduction

The ongoing trend toward lighter and more slender civil structures has increased their susceptibility to human-induced vibrations, particularly in pedestrian facilities such as footbridges, floors, and staircases. In this context, Human–Structure Interaction (HSI) has emerged as a critical research field, as the dynamic response of these systems is governed not only by structural properties but also by the mutual interaction between pedestrians and the supporting surface. Numerous experimental and numerical studies have shown that neglecting HSI effects may lead to inaccurate predictions of structural response, user comfort, and serviceability (Castillo et al., 2024a; Fouli and Camara, 2024; Mohammed and Pavic, 2021).

To address these challenges, increasingly sophisticated modeling strategies have been proposed (Berardengo et al., 2019; Caprani and Ahmadi, 2016; Castillo et al., 2024b; D'Hondt et al., 2024; Xie et al., 2021; Wang and Chen, 2022; Ippersiel et al., 2022; Dong and Noh, 2024; Emmerzaal et al., 2025), evolving from simplified moving-load representations toward biomechanical formulations in which the pedestrian is modeled as an articulated multi-segment system (Aux et al., 2024). These approaches aim to capture the coupled dynamics between

human motion and structural response more realistically.

In the multisegment biomechanical models of pedestrian walking, joint torques and ground reaction forces can be computed using inverse dynamics within an optimization-based framework (Singh and Lankarani, 2024; Ren et al., 2007). In these approaches, the motion of each body segment is prescribed a priori, and the corresponding joint moments are obtained by solving the equations of motion of the articulated system. The imposed kinematic patterns are not arbitrary; instead, they are determined as part of an optimization process in which the gait pattern of each multisegment model is identified by minimizing the mechanical energy expenditure of the walker, yielding motion trajectories that more closely resemble realistic human walking behavior.

The imposed segment displacements can be described using Fourier series, whose coefficients constitute the decision variables of the optimization problem (Ren et al., 2007). Biomechanical constraints are simultaneously enforced to ensure that joint rotations remain within physiologically admissible ranges, preventing non-biocompatible motions of the body segments. Consequently, the accuracy of the predicted pedestrian-induced forces strongly depends on the quality and generalizability of the resulting kinematic descriptions.

An accurate representation of human gait kinematics is therefore essential. However, human gait is a highly complex and adaptive biomechanical process (Kim et al., 2018) governed by the combined influence of intrinsic and extrinsic factors. Intrinsic parameters include subject-specific characteristics such as height, limb proportions, foot geometry, anthropometry, and self-selected walking speed, while extrinsic factors are associated with the mechanical properties of the supporting surface and environmental conditions (Multon et al., 2012; Šentija et al., 2012; Fukuchi et al., 2019a; Arellano-González et al., 2022; Villarroja-Aparicio and Pérez-García, 2003; Svenningsen et al., 2019; Gao et al., 2022). Variations in these parameters significantly affect spatiotemporal gait characteristics, joint kinematics, and load transmission mechanisms during walking (Gill et al., 2023; Jerome et al., 2015).

Within the kinematic chain, the effects of these intrinsic and extrinsic gait parameters are ultimately expressed through joint-level motion, which governs force transmission to the supporting surface. Among the lower-limb joints, the ankle plays a central role in regulating gait dynamics, contributing to propulsion, shock absorption, and balance control throughout the walking cycle (Lipfert et al., 2014; Zelik and Adamczyk, 2016; Crenna and Frigo, 2011). Changes in walking speed, anthropometry, or surface conditions directly influence ankle kinematics and kinetics, modifying joint angles, moments, and foot rollover behavior (Xie et al., 2021; Huang et al., 2025; Wang et al., 2025). Consequently, an accurate representation of ankle motion is essential for biomechanical gait models intended for HSI applications.

Due to its role as the initiating segment of the kinematic chain during walking, the foot is of particular importance in inverse dynamics formulations. Unlike other body segments, the foot does not exhibit a constant effective length, as its motion involves coordinated plantarflexion and dorsiflexion phases throughout the gait cycle. As a result, foot kinematics is typically introduced as an external input parameter rather than being optimized.

The foot kinematics depends on individual anthropometric characteristics and, therefore, its trajectory must usually be obtained through prior experimental measurements and subsequently imposed in the numerical model. Consequently, when simulating HSI for pedestrians with different anthropometric profiles and walking speeds using multisegment HSI models based on inverse dynamics and optimization, dedicated experimental tests are required for each subject prior to performing the numerical simulations. This requirement entails a substantial increase in experimental effort, time consumption, and associated costs, ultimately limiting the applicability of such models to large or diverse pedestrian populations.

Motivated by this limitation, the objective of the present study is to identify relationships between some intrinsic parameters and ankle kinematics during walking, enabling the description of ankle motion without requiring individual foot rollover measurements for each new subject. To this end, experimental ankle trajectories obtained from video-based motion processing at different walking speeds are analyzed, gait cycle phases are identified, and the influence of anthropometric pa-

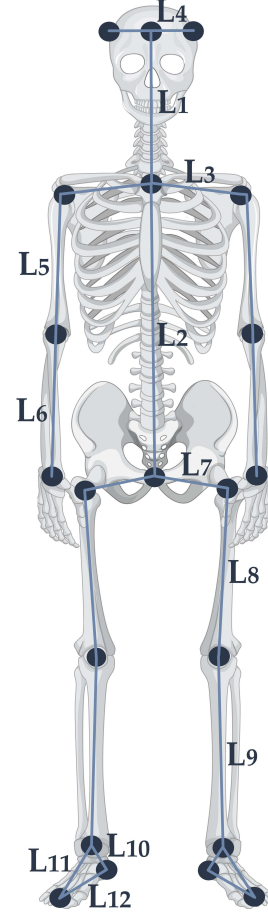


Figure 1: Anthropometric nomenclature.

rameters on ankle kinematics is systematically evaluated. The resulting formulation provides a scalable and generalizable representation of ankle motion that can be readily used into inverse dynamics and HSI analyses.

The overall methodological framework, including subject anthropometric characterization and the procedures for data collection and processing, is described in Section 2. Section 3 presents the results related to foot horizontal and vertical trajectories (X_{ank} and Y_{ank} , respectively), the calculated foot rollover angle (θ_{fr}), identified gait cycles, and the relationships established between these outcomes, subject anthropometry, and walking speed (v), together with the proposed model. Finally, Section 4 summarizes the main conclusions of the study.

2. Methods

2.1. Investigation General Methodology

Initially, an experimental campaign was conducted to collect ankle kinematic data using the Multi-axial Human–Structure Interaction Test Framework (HSI-MTF) (Castillo et al., 2024a) in combination with an opto-electrical motion capture system based on reflective markers and smartphone devices. The selection of this acquisition approach was guided by previous validation studies comparing its performance with commercial

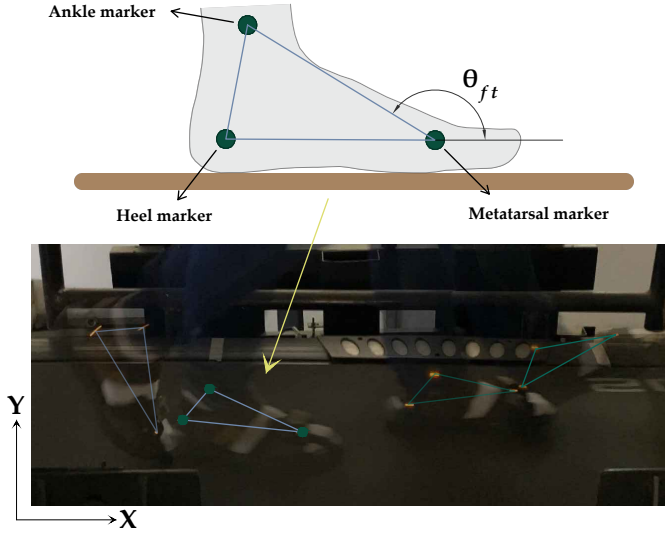


Figure 2: Markers placement.

motion capture systems such as OptiTrack (Nagymate and Kiss, 2018). Although commercial systems provide higher optical redundancy, they still present limitations related to marker visibility and spatial coverage. In contrast, the smartphone-based system, despite its inherent constraints in optical redundancy and spatial distribution, has been shown to deliver accurate kinematic measurements with small errors relative to commercial systems (Castillo et al., 2025).

4 females (F) and 6 males (M) participated as volunteers in this study. None of the participants reported any gait impairments or conditions affecting their normal walking pattern at the time of data collection. Detailed anthropometric data can be found in Table 1 (see the nomenclature in Figure 1). All research involving human subjects adhered strictly to the ethical standards outlined in the Declaration of Helsinki by the World Medical Association, and the study protocol was approved by the Ethics Committee at Universidad del Valle.

Initially, each test subject was equipped with three reflective markers attached to key points in the foot: ankle, heel, and metatarsus (Figure 2). For marker motion capture, video recording with smartphones at a sampling rate of 30 fps was conducted while subjects walked in 30-second test intervals at 6 v values (1.3 m/s - 1.6 m/s - 2.0 m/s - 2.2 m/s - 2.5 m/s - 3.0 m/s) on the level treadmill surface. The smartphones were strategically placed to provide a perpendicular view of the subject on the treadmill, ensuring a clear, unobstructed sagittal plane perspective. This setup allowed for precise tracking of the markers during gait, necessary for accurate sagittal coordinate analysis. Subsequently, through image processing, the marker coordinates were extracted from each frame to calculate θ_{ft} , at each instant of the test.

Subsequently, gait cycles were identified for each experimental trial to examine the influence of walking speed on individual gait patterns, with particular emphasis on foot plantarflexion during Toe Off (TO) and dorsiflexion during Heel Strike (HS) (Figure 3). This enabled the assessment and validation of

speed-dependent variations in ankle kinematics and their effects on both the linear and angular components of foot motion.

2.2. Kinematic data processing and modeling

The videos were processed using a custom MATLAB algorithm (R2023b) developed specifically for this study. The processing began by extracting each video frame sequentially. Within each frame, the algorithm performed image processing steps to identify and isolate individual markers.

To compute θ_{ft} , the ankle, heel and metatarsus coordinates of the identified markers were extracted from each video frame and processed through a vectorial formulation, using the dot product formula between two vectors:

$$\theta_{ft} = \arccos \left(\frac{\vec{MA} \cdot \vec{M}}{|\vec{MA}| |\vec{M}|} \right) \quad (1)$$

where $\vec{MA}$ is the foot longitudinal vector, defined from the metatarsal marker to the ankle marker, representing the orientation of the foot segment in the sagittal plane. The vector $\vec{M}$ is a reference vector defined from the metatarsal marker along the horizontal direction, representing the ground-aligned reference axis. This formulation allows the foot orientation to be quantified relative to the horizontal, enabling the identification of HS and TO phases the gait cycle. The MidStance phase (MS) was identified as the interval of sustained foot-ground contact, whereas HS and TO phases were determined from the local minima of the vertical coordinates of the heel and metatarsus markers, respectively.

Based on the experimental data, a numerical model was developed to describe ankle kinematics during walking. The foot rollover angle, θ_{ft} , is formulated as a function of the reference foot rollover angle during the stance phase (θ_{ft_0}) and is obtained from anthropometric foot measurements combined with a baseline profile scaled according to walking speed. Similarly, the horizontal and vertical ankle trajectories, X_{ank} and Y_{ank} , are described through Fourier series formulations coupled with scaling functions that account for the influence of walking speed and subject-specific anthropometric parameters.

Since several human-structure interaction models based on inverse dynamics and optimization, such as the formulation proposed by (Aux et al., 2024), consider only the ankle trajectory associated with the foot-ground contact phase where load transfer to the supporting surface occurs, the present study focuses exclusively on the portion of the ankle trajectory corresponding to the stance phase.

3. Results and Discussion

3.1. Influence of v on ankle kinematics

In Figure 4, the processed trajectories of the ankle marker coordinates from one of the test subjects are presented. An increase in the amplitude of the horizontal displacement is observed with increasing v values, indicating a progressive lengthening of the stride. This behavior is consistent with previous studies identifying stride length as a kinematic variable strongly

Table 1
Anthropometric information.

		Test Subject									
		1 (M)	2 (M)	3 (M)	4 (M)	5 (F)	6 (F)	7 (M)	8 (F)	9 (F)	10 (M)
General Information	Age	21.0	22.0	23.0	23.0	23.0	24.0	23.0	21.0	22.0	22.0
	Height (cm)	167.0	178.0	176.0	165.0	167.0	170.0	174.0	163.0	163.0	180.0
	Weight (Kg)	47.70	87.90	90.00	71.00	46.50	98.30	71.40	49.40	60.00	70.70
	BMI	17.10	27.74	29.05	26.08	16.67	34.01	23.58	18.59	22.58	21.82
Perimeters (cm)	Head	59.00	57.00	58.00	59.30	59.00	59.00	59.00	54.50	57.50	57.00
	Shoulder	35.50	42.00	41.00	42.00	33.00	40.00	44.00	35.00	40.00	43.50
	Elbow	23.00	27.00	26.00	25.50	23.00	28.00	27.00	22.00	23.50	26.00
	Wrist	16.00	17.00	17.00	17.00	15.00	17.50	17.00	14.50	15.00	16.50
	Hip	70.00	102.00	104.00	111.50	79.00	110.00	81.00	80.00	100.50	84.00
	Knee	36.00	44.00	43.00	36.50	33.00	45.50	41.00	39.00	35.00	41.00
	Ankle	23.00	26.00	26.00	26.00	23.00	25.00	27.00	21.50	24.50	26.00
	Metatarsal	20.50	22.00	22.50	22.70	21.00	24.00	23.00	19.50	19.00	23.50
	Heel	20.00	28.00	28.00	25.00	20.50	27.00	24.00	23.00	21.00	23.00
Lengths (cm)	L_1	25.00	27.00	27.50	24.50	23.00	25.00	26.00	23.00	22.50	25.00
	L_2	48.00	49.00	48.00	46.70	47.00	46.00	43.00	43.00	50.00	47.00
	L_3	22.00	23.00	23.00	19.00	19.00	20.00	22.00	18.50	23.00	22.00
	L_4	8.00	8.00	7.50	8.00	7.00	7.00	7.00	7.50	8.50	7.50
	L_5	25.00	33.00	32.00	28.00	25.00	26.00	28.00	29.00	30.00	31.00
	L_6	26.00	27.00	27.00	25.50	26.00	25.50	26.00	26.00	25.00	29.00
	L_7	15.00	19.00	19.00	21.00	17.00	23.00	15.00	15.00	25.00	17.00
	L_8	49.00	52.00	51.00	56.00	52.50	52.00	54.00	51.00	52.00	55.00
	L_9	40.00	42.00	41.50	40.50	40.00	44.00	42.00	42.00	39.00	45.00
	L_{10}	7.50	11.00	11.00	7.00	7.00	8.00	10.00	8.00	11.00	10.00
	L_{11}	20.00	22.00	22.00	19.00	21.00	21.00	22.00	21.00	19.50	22.50
	L_{12}	23.00	27.50	27.00	22.50	25.00	26.50	26.00	24.00	23.00	27.50

influenced by gait velocity (Fukuchi et al., 2019a; Gill et al., 2023; Jerome et al., 2015). Likewise, the maximum amplitude of the vertical ankle displacement over the gait cycle follows the same trend: at a slow value of 1.0 m/s, the vertical amplitude is reduced because the mechanics of the ankle joint are characterized by a smaller range of motion and limited power generation during TO. Previous studies have reported that, at slower speeds, the amplitude of the trajectories tends to decrease, reflecting a lower overall mechanical demand (Fukuchi et al., 2019b). In this context, the plantar-flexor muscles of the ankle generate less positive power during TO, thus reducing both forward propulsion and vertical elevation of the foot (Lipfert et al., 2014; Zelik and Adamczyk, 2016). In contrast, at higher v values such as 2.0 m/s and 3.0 m/s, a greater vertical ankle amplitude is observed. This occurs because ankle dorsiflexion during the HS plays a crucial role in increasing foot clearance from the ground, thus allowing a higher trajectory (Bajelan et al., 2024). Moreover, TO under these conditions produces a burst of positive power that simultaneously propels the center of mass and accelerates the swing leg (Zelik and Adamczyk, 2016), acting as a catapult-like mechanism that enhances vertical elevation (Lipfert et al., 2014).

Furthermore, as v increases, the gait cycle frequency also rises. This effect can be observed through the progressive shortening of the stance and swing phases with increasing v values,

reflecting the reduced time available for each phase of the gait cycle. Consequently, the temporal structure of the gait becomes more dynamic, with quicker transitions between HS and TO events, which is consistent with the mechanical adaptations required to maintain faster locomotion.

In Figure 5, the evolution of θ_{fi} for one of the test subjects is shown for different values of v . At lower v , the rollover angle at HS is reduced, indicating a diminished dorsiflexion requirement at initial contact, while during TO the plantarflexion demand is also lower, as this event occurs at comparatively larger values of θ_{fi} under slow walking conditions. Together, these features indicate a reduced overall range of motion of the ankle-foot complex at low v , in agreement with previous studies reporting smaller joint excursions and decreased mechanical demand during slow gait. In contrast, as v increases, the range of motion of the rollover angle progressively expands, characterized by increased dorsiflexion at HS and more pronounced plantarflexion during TO. This speed-dependent behavior is consistent with biomechanical evidence showing that higher walking speeds are associated with increased ankle power generation and enhanced push-off mechanics, which contribute to greater forward propulsion and improved foot clearance (Fukuchi et al., 2019b; Lipfert et al., 2014; Zelik and Adamczyk, 2016).

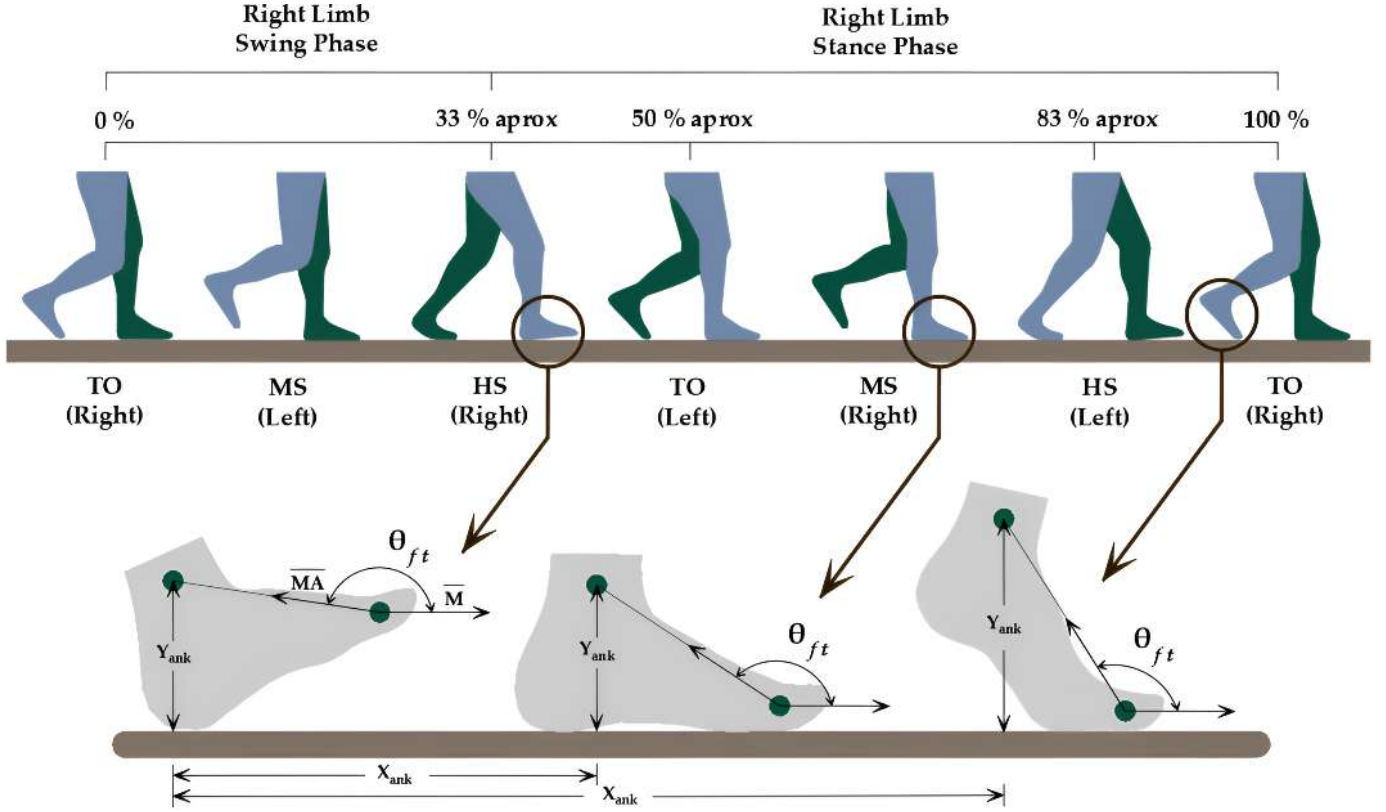


Figure 3: Complete gait cycle and foot rollover representation. The figure illustrates the complete gait cycle of the right lower limb, beginning at 0% with TO of the right foot, which marks the start of the swing phase. During this interval, the left limb simultaneously undergoes its MS phase, ensuring stability and support. At approximately 33%, the right foot performs HS, initiating the stance phase, which continues until the subsequent TO (100%). Throughout this phase, the right limb supports body weight, contributes to balance, and generates the propulsion required for forward progression, while the contralateral limb alternates in the opposite phase.

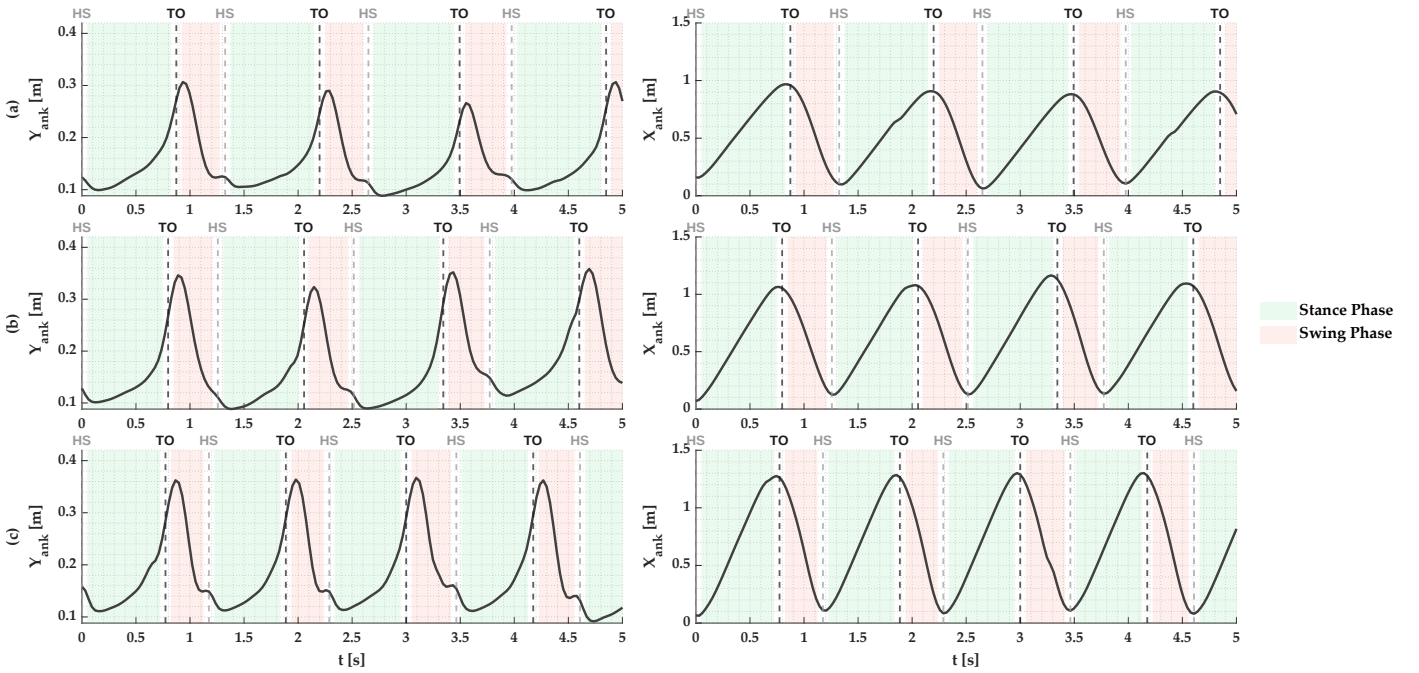


Figure 4: Displacements of X_{ank} and Y_{ank} during a 5-second interval of the experimental test, for three walking speeds: (a) 1.3, (b) 2.0, and (c) 3.0 m/s. The characteristic gait cycle instants HS and TO are indicated, delimiting the stance and swing phases for each trajectory.

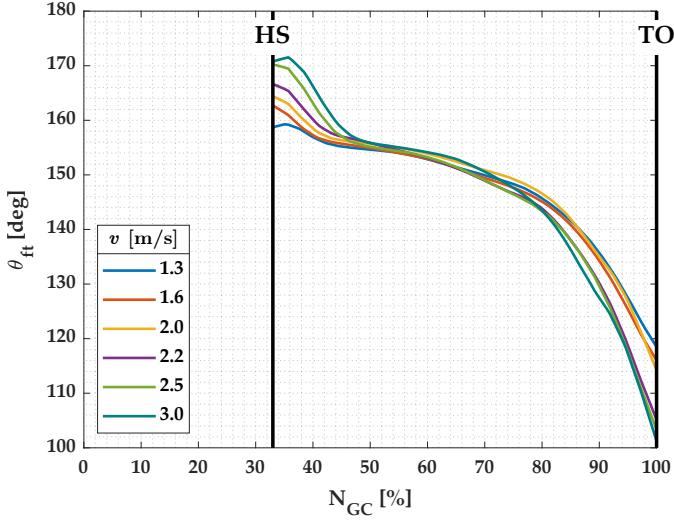


Figure 5: Variation of the right θ_{ft} according to the normalized gait cycle (N_{GC}) during the stance phase, from HS to TO, at different v values. The curves represent the mean values of each test for the first subject at each evaluated v .

3.2. Influence of anthropometry on ankle kinematics

Additionally, a strong influence of subject anthropometry on ankle kinematics was confirmed. During the development of the parametric models described in Section 3.4, several anthropometric variables were evaluated in order to identify those providing the best predictive performance. The results consistently indicated that subject height and total leg length ($L_8 + L_9$), exhibited the strongest correlation with ankle kinematic trajectories. Models incorporating these parameters yielded the lowest fitting errors and the most stable representations across subjects at different v values.

3.3. Intra-subject variability of ankle kinematics

To provide a more detailed insight into ankle joint behavior, Figure 6 presents the relationship between the ankle displacements $X_{ank}-Y_{ank}$ and the foot rollover angle θ_{ft} for one representative subject at a given value of v . This representation shows that the temporal evolution of θ_{ft} is accompanied by coordinated changes in both horizontal and vertical ankle trajectories, resulting in a smooth and repeatable kinematic pattern across consecutive gait cycles. Small cycle-to-cycle variations are observed, which are more pronounced in the horizontal ankle displacement than in the vertical component. These differences reflect the inherent variability of step length and forward progression between cycles, while the vertical motion remains comparatively consistent. Although subtle, such variations are relevant, as neglecting them may lead to an oversimplified description of ankle kinematics. Accounting for this variability, especially in the horizontal trajectory, contributes to a more realistic representation of gait and enhances the robustness of biomechanical and HSI models under varying walking conditions.

3.4. Intrinsic parameters-based model

Ankle kinematics can be incorporated into the HSI modeling framework through the development of numerical models that

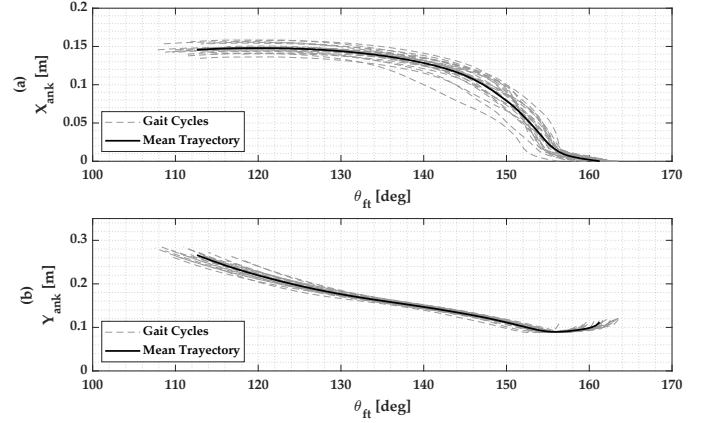


Figure 6: Gait pattern of subject 1 at a walking speed of 1.6 m/s during the stance phase of the right limb: (a) X_{ank} and (b) Y_{ank} . The plots display the behavior of each gait cycle, which in this case corresponds to 18 cycles recorded over the 30-second test, together with the mean trajectory of each displacement.

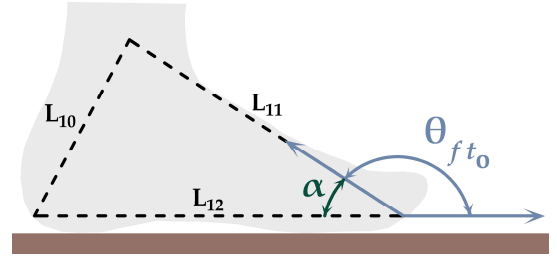


Figure 7: Schematic representation of θ_{ft0} .

provide a consistent basis for reproducing subject-dependent motion patterns. These models are formulated by integrating the relationships identified through experimental analysis into a mathematical formulation that enables the direct computation of both angular and linear ankle trajectories. In this section, the proposed models are described.

3.4.1. θ_{ft} numerical model

For the development of the proposed model, it is initially assumed that the foot rollover angle during the MS phase (Figure 7) can be estimated from subject-specific foot anthropometric parameters through trigonometric relationships:

$$\theta_{ft0} = 180 - \alpha \quad [\text{deg}] \quad (2)$$

where the angle α is defined as:

$$\alpha = \arccos\left(\frac{L_{10}^2 - L_{11}^2 - L_{12}^2}{-2L_{11}L_{12}}\right) \quad [\text{deg}] \quad (3)$$

This value is established as the subject-specific reference for the model. Based on this, θ_{ft} trajectories corresponding to different v values are normalized, enabling its progression during the stance phase to be quantified as a percentage of its MS value (θ_{ft0}).

Based on this formulation, the model estimates the θ_{ft} trajectory by defining its normalized evolution relative to the MS

value. This trajectory is subsequently scaled by a subject-specific midstance angle, derived from anthropometric parameters, to reconstruct the full θ_{ft} curve during the stance phase. The model's general mathematical formulation is given by:

$$\theta_{ft} = \left(\frac{\theta_{ft}}{\theta_{ft_0}} \right)^* \theta_{ft_0} \quad (4)$$

where the normalized foot rollover angle is defined as:

$$\left(\frac{\theta_{ft}}{\theta_{ft_0}} \right)^* = \begin{cases} f(N_{GC}) \frac{f_{HS}(v) - 1}{0.035}, & N_{GC} \leq 54, \\ f_{TO}(v) - \left(\frac{f(N_{GC}) - 0.770}{0.230} \right) (1 - f_{TO}(v)), & N_{GC} > 54. \end{cases} \quad (5)$$

This function is based on a baseline curve $f(N_{GC})$, represented by a third-order Fourier series that captures the general shape of the foot rollover angle trajectory (Eq. (6)). The coefficients of all Fourier series introduced in this section are reported in Table 2.

$$f(N_{GC}) = a_0 + \sum_{i=1}^3 [a_i \cos(i\omega N_{GC}) + b_i \sin(i\omega N_{GC})] \quad (6)$$

This curve is scaled according to v through two regression-based functions that describe the percentage of θ_{ft} during the HS and TO phases:

$$f_{HS}(v) = 0.016v + 1.048 \quad (7)$$

$$f_{TO}(v) = -0.034v + 0.770 \quad (8)$$

3.4.2. X_{ank} numerical model

Eq. (9) presents the general formulation of the proposed numerical model for X_{ank} trajectory. The model is based on a collapsed, dimensionless representation obtained by normalizing the experimental ankle trajectories of all subjects. The normalization process is performed by dividing the original ankle curve by two scaling functions: a velocity-dependent function, $f_x(v)$, and an anthropometry-dependent function, $f_x(h)$. This procedure approximates a collapsed curve that represents the shape of the horizontal ankle motion across different subjects and walking speeds. The collapsed trajectory is expressed as a numerical function obtained from a Fourier series representation (Eq. (10)). Once defined, the ankle trajectory can be reconstructed for a desired v value and subject anthropometry by scaling the collapsed curve using the same functions $f_x(v)$ and $f_x(h)$.

$$X_{ank} = \left(\frac{X_{ank}}{f_x(v)f_x(h)} \right)^* f_x(v) f_x(h) \quad (9)$$

$$\left(\frac{X_{ank}}{f_x(v)f_x(h)} \right)^* = a_0 + \sum_{i=1}^2 a_n \cos(i\omega f(N_{GC})) + b_n \sin(i\omega f(N_{GC})) \quad (10)$$

The mathematical definitions for $f_x(v)$ and $f_x(h)$ are provided in Eqs. (11) and (12), respectively. The exponents γ and β (Eqs. 13 and 14) establish a link between the subject's height (h) and the background curve of the θ_{ft} model, $f(N_{GC})$, ensuring that the scaling factor adapts dynamically throughout the gait cycle.

$$f_x(v) = v^{-0.150} + v^{0.700} \quad (11)$$

$$f_x(h) = 0.250^{\gamma+\beta} \quad (12)$$

$$\gamma = 0.660f(N_{GC}) - 0.350 \left| \frac{h}{1.700} - 1 \right|^{0.850} \quad (13)$$

$$\beta = -0.800f(N_{GC}) - 0.700 \left| \frac{h}{1.700} - 1 \right|^{0.850} \quad (14)$$

3.4.3. Y_{ank} numerical model

Eq. (15) defines the numerical model for the Y_{ank} trajectory. Although the formulation follows the same normalization and scaling framework adopted for X_{ank} , only anthropometric variability is considered in this case. This simplification is justified by the observation that walking speed has a relatively minor influence on the vertical displacement of the ankle during the stance phase. The Y_{ank} trajectory is normalized using the anthropometry-dependent function $f_y(L_8, L_9)$, which enables a collapsed representation that captures a common vertical ankle motion pattern across subjects. Among the anthropometric parameters evaluated, leg length ($L_8 + L_9$) showed the strongest correlation with the vertical ankle trajectory.

$$Y_{ank} = \left(\frac{Y_{ank}}{f_y(L_8, L_9)} \right)^* f_y(L_8, L_9) \quad (15)$$

The scaling function $f_y(L_8, L_9)$, defined in Eq. (16), is therefore expressed in terms of leg length. The parameters v and δ (Eqs. (17)-(18)), incorporate the dependence on N_{GC} .

$$f_y(L_8, L_9) = 0.090^{0.250v+0.750\delta} \quad (16)$$

$$v = 0.050N_{GC} \left(\frac{0.940}{L_8 + L_9} \right) \quad (17)$$

$$\delta = 9e^{-4}N_{GC} \left(\frac{0.940}{L_8 + L_9} \right) \quad (18)$$

3.5. Model validation against experimental data

Figures 8–10 compare the proposed numerical models against experimental data for Subject 1 across various v values. The models demonstrate strong agreement with experimental measurements, accurately capturing the primary trends in ankle kinematics during the stance phase. Quantitatively, the θ_{ft} trajectory achieved a high precision with an average relative error of 1.007%. X_{ank} and Y_{ank} displacements yielded average errors of 9.359% and 5.544%, respectively, confirming the fidelity of the angular component and an acceptable predictive performance for spatial trajectories, validating the robustness of the proposed modeling approach.

Table 2 Values of the Fourier series coefficients corresponding to the background curves of each model.

Function	w	a_0	a_1	b_1	a_2	b_2	a_3	b_3
$\left(\frac{\theta_{ft}}{\theta_{ft0}}\right)^*$	0.0559	0.9185	-0.1230	0.0546	-0.0611	0.0176	-0.0169	-0.0003
$\left(\frac{X_{ank}}{f_x(v)f_x(h)}\right)^*$	13.9858	0.0320	0.0223	-0.0240	-0.0020	0.0091	—	—
$\left(\frac{Y_{ank}}{f_y(L_8, L_9)}\right)^*$	9.7679	5.4938	5.4938	0.8788	0.9491	-0.7182	—	—

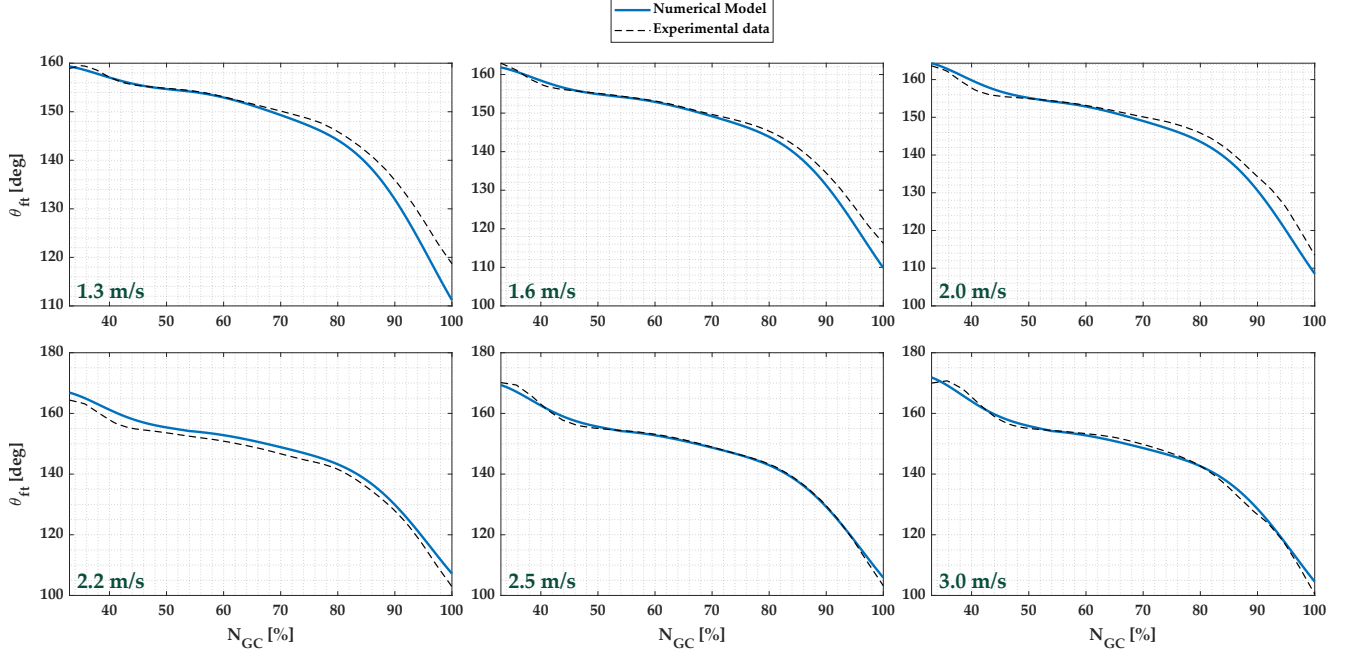


Figure 8: Comparison of the θ_{ft} trajectory during the gait cycle between the proposed numerical model and the experimental data for Test Subject 1 at different v values.

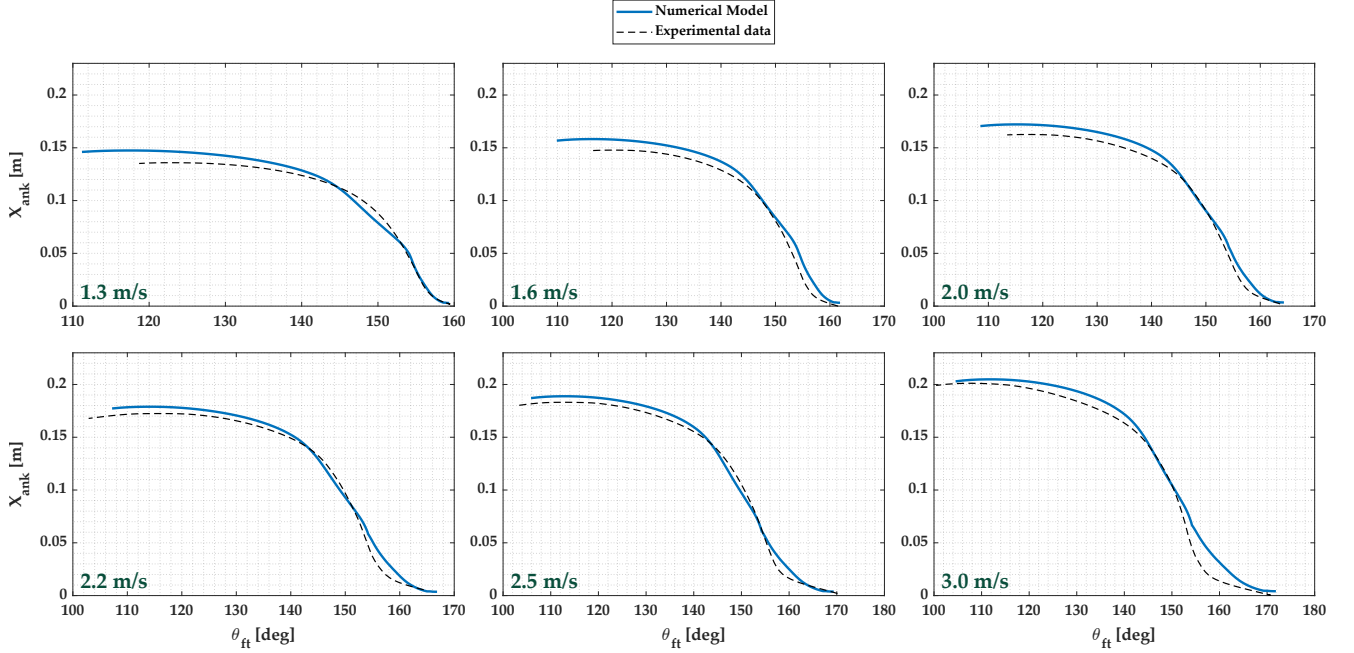


Figure 9: Comparison between X_{ank} vs θ_{ft} obtained from the proposed numerical model and the corresponding relationship derived from experimental data, for Test Subject 1 at different v values..

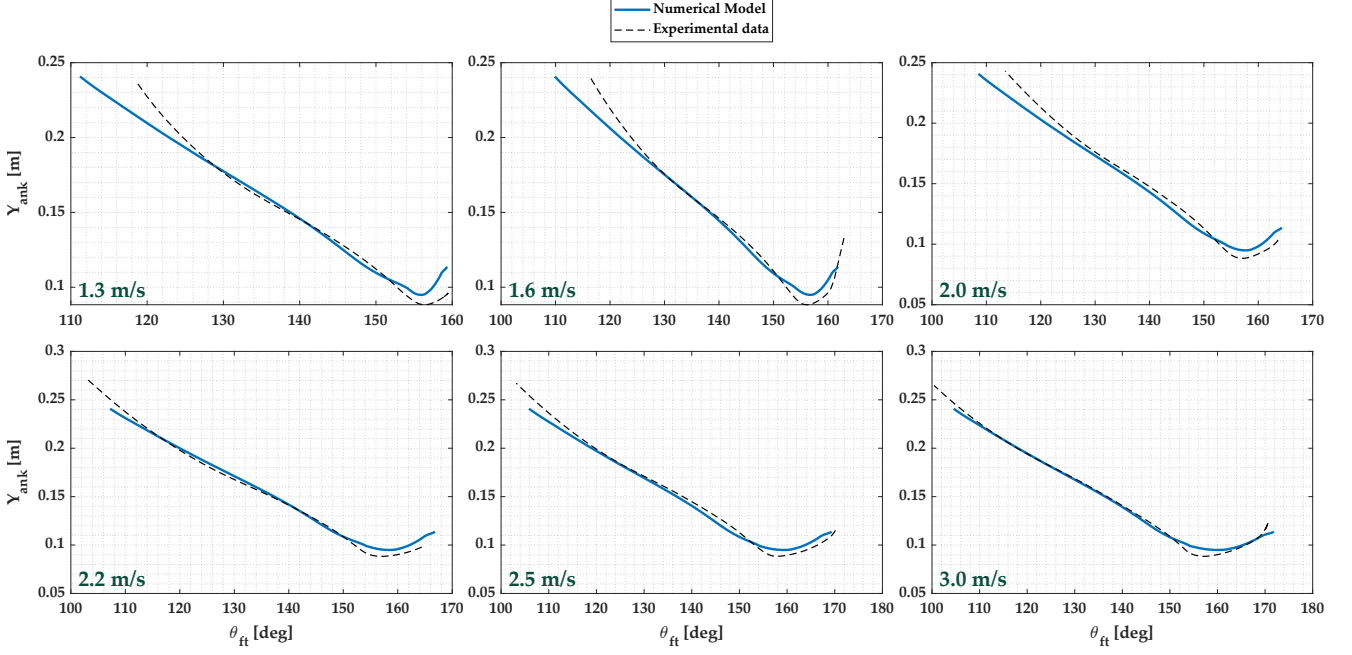


Figure 10: Comparison between Y_{ank} vs θ_{ft} obtained from the proposed numerical model and the corresponding relationship derived from experimental data, for Test Subject 1 at different v values.

3.6. Incorporation of intra-subject variability into the model

As established in Section 3.3, ankle kinematics exhibit smooth and repeatable patterns across consecutive gait cycles. Nevertheless, measurable inter-cycle variability persists, particularly in the horizontal direction. To incorporate this inherent dispersion and avoid an overly simplified representation of the motion, confidence bands have been introduced into the proposed kinematic models.

These bands are constructed by proportionally varying the coefficients of the Fourier series that define the background curve of each model. The upper and lower bounds are determined such that the experimental data, normalized with respect to the functions presented in Sections 3.4.2 and 3.4.3, are fully contained within the corresponding bands. The analysis reveals that a variation of $\pm 10\%$ in the Fourier coefficients is sufficient for the X_{ank} model, whereas the Y_{ank} model requires a wider range of $\pm 20\%$ (Figure 11).

To capture cycle-to-cycle variability, the coefficients of each model are allowed to fluctuate within their respective confidence bands. Consequently, the background curve is not identical in every cycle, resulting in small but realistic variations in the ankle trajectory.

4. Conclusions

The present study proposes a parametric representation of ankle kinematics based on intrinsic gait parameters for application in inverse dynamics-based HSI models. Experimental results show that walking speed and anthropometric characteristics have a direct influence on ankle motion. In particular, subject height and total leg length were identified as relevant

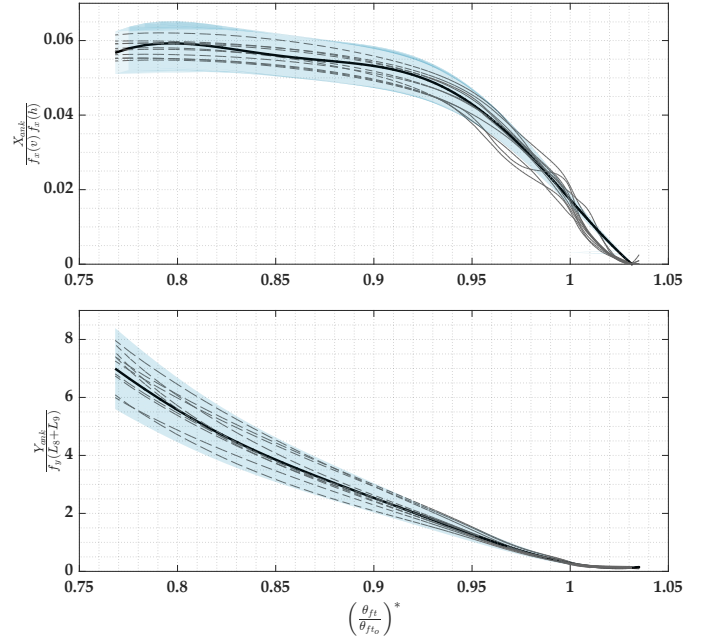


Figure 11: Confidence bands associated with the background curves of the X_{ank} and Y_{ank} models, obtained from the normalization of the experimental data of the test subjects. The dashed lines represent the individual normalized curves, while the solid black line denotes the mean response, defined as the background curve fitted using Fourier series from the normalized data.

parameters for describing horizontal and vertical ankle trajectories, respectively.

Intra-subject variability was addressed by introducing confidence bands into the proposed formulation, allowing the model to reproduce cycle-to-cycle variations observed in the experimental data without relying on deterministic trajectories. The

comparison with experimental measurements showed low relative errors, indicating that the proposed models provide an accurate and computationally efficient description of ankle kinematics during the stance phase.

The main limitation of the current formulation lies in the simplified representation of foot motion, as only sagittal-plane kinematics are considered. Future developments should incorporate the frontal inclination of the foot, yielding a three-dimensional approximation of foot trajectories and a more complete description of ankle-foot mechanics.

Finally, further validation is required using a more diverse population, including individuals with conditions that affect gait dynamics. Such studies would allow assessing the robustness of the model under altered walking patterns and guide future improvements for broader HSI applications.

CRedit authorship contribution statement

Marcela Perlaza – Conceptualization; Methodology; Investigation; Formal analysis; Software; Validation; Visualization; Writing – original draft; Writing – review & editing.

Bryan Castillo – Conceptualization; Supervision; Writing – review & editing.

Peter Thomson – Conceptualization; Supervision; Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. All authors have contributed intellectually to the submitted manuscript in terms of content, organization, and presentation. This paper is an original work and has not been submitted elsewhere for consideration.

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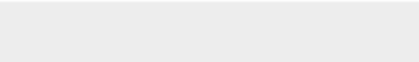
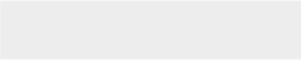
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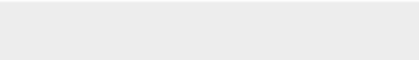
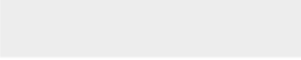
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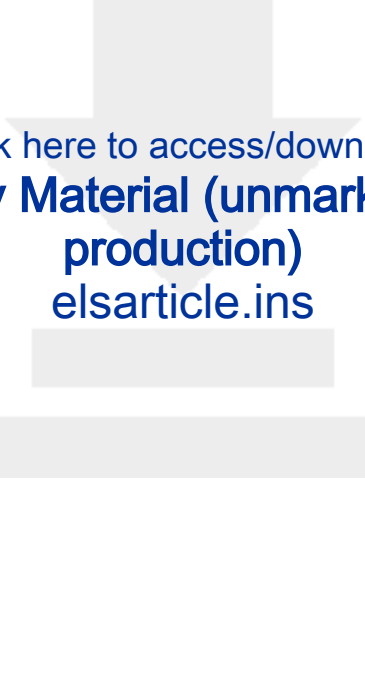




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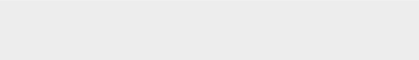
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Parametric Characterization of Ankle Kinematics for Inverse Dynamics–Based Human–Structure Interaction Models

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