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Free convection effects on mhd flow past an infinite vertical accelerated plate embedded in porous media with constant heat flux

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Abstract

This paper analyses the free convection flow of a viscous incompressible fluid past an infinite vertical accelerated plate embedded in porous medium with constant heat flux in the presence of transverse magnetic field. The governing equations are solved in closed form by Laplace-transform technique. The results are obtained for velocity, temperature, skin-friction and Nusselt number. The effects of M-magnetic field parameter, Pr-Prandtl number, G-Grashof number, K-permeability parameter are discussed on the flow variables and presented by graph.

Keywords: Free convection, Magnetohydrodynamic flow, Porous medium, Heat flux

MSC(2000): 76R10, 76D05, 76W05, 76S05, 44A10

1 Introduction

Free convection flows are frequently encountered is physical and engineering problems such as chemical catalytic reactors, nuclear waste materials, geothermal systems etc. The free convection flow past a vertical plate studied by Kolar et al. [1] and Ramanajah et al. [2] with different boundary conditions. The problem of natural convective cooling of a vertical plate solved numerically by Camargo et al. [3].

Extensive research work published on an impulsively started vertical plate. Stokes [4] first presented an exact solution to the Navier-Stokes equation for flows past an impulsively started infinite horizontal plate. Instead of horizontal plate, if an infinite isothermal vertical plate is given an impulsive motion, how the flow is affected by the free convection currents, which exists due to temperature difference between the plate temperature and that of fluid far away from the plate? This was first studied by Soundalgekar [5] who presented an exact solution for free convection effects on the Stokes problem for an infinite vertical plate. By applying magnetic field in flow past an impulsively started vertical plate investigated by Raptis et al. [6] and Murty [7]. Unsteady free convection flow past an impulsively started plate considered by Makinde et al. [8].

MHD flows have many applications, for example in MHD electrical power generation geophysics etc. MHD flows are found in many industrial applications. Moreau [9] listed some of these numerous industrial applications. Gebhart et al. [10] indicated that early interest in such flows arose in astrophysics, geophysics and controlled nuclear physics. The study of the effect of magnetic field on free convection flows is important in liquid-metals, electrolytes and ionized gases. Several MHD free convection solutions were presented by Cramer [11]. MHD free convection flow past a vertical plate investigated by researchers like, Gupta [12], Pop [13], Sahoo et al. [14] and Sharma et al. [15] with different boundary conditions.

Porous medium has its applications such as those involving heat removal from nuclear fuel debris, under ground disposal of radio active waste material, storage of food stuffs etc. Furthermore convection through porous media may be found in fiber and granular insulation. Representative studies in this area may be found in books by Nield et al. [16], Bejan et al. [17], Ingham et al. [18]. Kim et al. [19] and Harris et al. [20] analyzed the free convection flow past a vertical plate in porous medium with different boundary conditions. Recently, Magyari et al. [21] discussed analytical solutions for unsteady free convection in porous media. The effect of MHD in porous media considered by Raptis et al. [22] and Geindreau et al. [23].

In many problems particularly those involving the cooling of electrical and nuclear components, the wall heat flux is specified. In such problems, over heating burns out and melt down are very important issues. From practical stand point, an important wall model is considered with constant heat flux. The flow through a porous medium bounded by a vertical infinite surface with constant heat flux investigated by Sharma [24]. An exact solution for MHD free convection flow with constant surface heat flux is obtained by Sacheti et al. [25].

It is proposed here to study the free convection effects on MHD flow past an infinite vertical accelerated plate embedded in porous media with constant heat flux by Laplace-transform technique for small values of magnetic parameter.

2 Formulation of the problem

We consider a two-dimensional flow of an incompressible electrically conducting viscous fluid along an infinite non-conducting vertical flat plate through a porous medium. Initially the plate and the fluid are at some temperature T'_∞ in a stationary condition. The x'-axis is taken along the plate in the vertically upward direction and the y'-axis is taken normal to the plate. At time $t' > 0$ a magnetic field of uniform strength is applied in the direction of y'-axis and the induced magnetic field is neglected. At time $t' > 0$, the plate starts moving impulsively in its own plane with a velocity U_0 with heat supplied to the plate at constant rate.

The governing equations of motion and energy under usual Boussinesq's approximation are given by:

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y^2} \tag{1}$$

and

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y^2} - \frac{\sigma B_0^2 u'}{\rho} + g\beta(T' - T'_\infty) - \frac{\nu u'}{K'} \tag{2}$$

With the following initial and boundary conditions:

$$u' = 0, \quad T' = T'_\infty, \quad y', t' \leq 0 \tag{3}$$

$$\begin{aligned} u' = U_0, \ \frac{\partial T'}{\partial y'} = -\frac{q'}{\kappa}, \quad y' = 0, t' > 0 \\ u' = 0, \quad T' = T'_\infty, \quad y' \to \infty, t' > 0 \end{aligned} \tag{4}$$

Introducing the following dimensionless quantities:

$$\begin{aligned} &u = \frac{u'}{U_0}, \quad t = \frac{t'U_0^2}{\nu}, \quad y = \frac{y'U_0^2}{\nu}, \quad M = \frac{\sigma B_0^2 \nu}{\rho U_0^2}, \quad Pr = \frac{\mu C_p}{\kappa}, \\ &K = \frac{U_0^2 K'}{\nu^2}, \quad \theta = \frac{T' - T'_\infty}{q'\nu / \kappa U_0}, \quad G = \frac{g\beta q' \nu^2}{\kappa U_0^4}, \end{aligned} \tag{5}$$

in equations (1) and (2), we get

$$Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \tag{6}$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G\theta - Mu - \frac{u}{K} \tag{7}$$

with the following initial and boundary conditions

$$u = 0, \ \ \theta = 0, \ \ y, t \leq 0 \tag{8}$$

$$\begin{aligned} u = 1, \ \frac{d\theta}{dy} = -1, \ \ y = 0, \ t > 0 \\ u = 0, \quad \theta = 0, \quad y \to \infty, \ t > 0 \end{aligned} \tag{9}$$

All the physical variables are defined in the nomenclature.

3 Method of solution

We solve the governing equation in exact form by Laplace transform technique. The Laplace transform of the equations (6), (7) and boundary conditions (9) are given by

$$\frac{d^2 \bar{\theta}}{dy^2} - pPr\bar{\theta} = 0 \tag{10}$$

$$\frac{d^2 \bar{u}}{dy^2} - (M' + p)\bar{u} = -G\bar{\theta} \quad \text{with } M' = M + \frac{1}{K} \tag{11}$$

$$\begin{aligned} \bar{u} = \frac{1}{p}, \ \frac{d\bar{\theta}}{dy} = -\frac{1}{p}, \quad y = 0, \ t > 0 \\ \bar{u} = 0, \quad \bar{\theta} = 0, \quad y \to \infty, \ t > 0 \end{aligned} \tag{12}$$

Solving equations (10), (11) with the help of equation (12), we get

$$\bar{\theta}(y,p) = \frac{1}{\sqrt{Pr}p^{3/2}} e^{-\sqrt{Prp}y} \tag{13}$$

$$\bar{u}(y,p) = \frac{e^{-y\sqrt{(p+M)}}}{p} + \frac{Ge^{-y\sqrt{(p+M)}}}{p^{3/2}\sqrt{Pr}(p(Pr-1)-M')} - \frac{Ge^{-y\sqrt{(Pr\,p)}}}{p^{3/2}\sqrt{Pr}(p(Pr-1)-M')} \tag{14}$$

On taking inverse Laplace transform of equations (13), (14), we get

$$\theta = \frac{2\sqrt{t}}{\sqrt{Pr}} \left\{ \frac{1}{\sqrt{\pi}} \exp(-\eta^2 Pr) - \eta\sqrt{Pr}\ \mathrm{erfc}(\eta\sqrt{Pr}) \right\} \tag{15}$$

For $Pr \neq 1$

$$\begin{aligned}
u(y,t) =& \frac{1}{2}\exp(2\eta\sqrt{M't})\mathrm{erfc}(\eta + \sqrt{M't}) + \frac{1}{2}\exp(-2\eta\sqrt{M't})\mathrm{erfc}(\eta - \sqrt{M't}) \\
&+ \frac{\mathrm{G}}{\pi M'\sqrt{Pr}} \left\{ \exp(ct) \int_0^\infty \frac{\cos y\sqrt{x}(1-\exp(-t(x+M'+c)))}{(x+M'+c)\sqrt{x+M'}} \mathrm{dx} \right. \\
&- \int_0^\infty \frac{\cos y\sqrt{x}(1-\exp(-(t(x+M'))))}{x+M'^{3/2}} \\
&- \int_0^{M'} \frac{exp(-y\sqrt{x+M'})(1-\exp(-xt))}{x^{3/2}} dx \\
&\left. + \exp(ct) \int_0^{M'} \frac{\exp(-y\sqrt{x+M'})(1-\exp(-t(x+c)))}{(x+c)\sqrt{x}} dx \right\} \\
&+ \frac{G}{M'\sqrt{Pr}} \left\{ \frac{2\sqrt{t}}{\sqrt{\pi}} \exp(-\eta^2 Pr) - 2\eta\sqrt{Pr\,t}\,\mathrm{erfc}(\eta\sqrt{Pr}) \right\} \\
&- \frac{G\exp(ct)}{2M'\sqrt{c\,Pr}} \left\{ \exp(-2\eta\sqrt{c\,Pr\,t})\,\mathrm{erfc}(\eta\sqrt{Pr} - \sqrt{ct}) \right. \\
&\left. - \exp(2\eta\sqrt{c\,Pr\,t})\,\mathrm{erfc}(\eta\sqrt{Pr} + \sqrt{ct}) \right\} \quad \text{Where} \quad c = \frac{M'}{Pr-1}
\end{aligned} \tag{16}$$

And for Pr $= 1$

$$
\begin{aligned}
u(y,t) =& \frac{1}{2}\exp(2\eta\sqrt{M't})\,\mathrm{erfc}\,(\eta + \sqrt{M't}) \\
&+ \frac{1}{2}\exp(-2\eta\sqrt{M't})\mathrm{erfc}\,(\eta - \sqrt{M't}) \\
&+ \frac{G}{\pi M'}\left\{-\int_0^\infty \frac{\cos y\sqrt{x}\,(1-\exp(-t(x+M')))}{(x+M')^{3/2}}\right. \\
&\qquad \left. - \int_0^{M'} \frac{\exp(-y\sqrt{x+M'})(1-\exp(-xt))}{x^{3/2}}dx\right\} \\
&+ \frac{G}{M'}\left\{\frac{2\sqrt{t}}{\sqrt{\pi}}\exp(-\eta^2) - 2\eta\sqrt{t}\mathrm{erfc}(\eta)\right\} \qquad (17)
\end{aligned}
$$

where $\eta = \dfrac{y}{2\sqrt{t}}$

We have computed numerical values of u by separating terms containing c into real and imaginary parts. The integrals are evaluated numerically. In expression, $\mathrm{erfc}(x+iy)$ is complementary error function of the complex argument which can be calculated in terms of tabulated functions [26]. The tables given in [26] do not give $\mathrm{erfc}(x+iy)$ directly but an auxiliary function $W_1(x+iy)$ which is defined as

$$\mathrm{erfc}(x+iy) = W_1(-y+ix\exp(-(x+iy)^2))$$

some properties of $W_1(x+iy)$ are

$$
\begin{aligned}
W_1(-x+iy) &= W_2(x+iy) \\
W_1(x-iy) &= 2\exp(-(x+iy)^2) - W_2(x+iy)
\end{aligned}
$$

where $W_2(x+iy)$ is complex conjugate of $W_1(x+iy)$.

Skin-friction

We now study the skin-friction from velocity field. It is given by

$$\tau = -\mu\frac{\partial u'}{\partial y'}\bigg|_{y'=0}$$

which in virtue of (5) reduces to

$$\tau = \frac{\tau'}{\rho U_0^2} = -\frac{du}{dy}\bigg|_{y=0}$$

For $Pr \neq 1$

$$\tau = \sqrt{M'}\mathrm{erf}(\sqrt{M't}) + \frac{\exp(-M't)}{\sqrt{\pi t}} - \frac{G}{\pi M'\sqrt{Pr}}\left\{\int_0^{M'} \frac{\sqrt{(x+M')}(1-\exp(-xt))}{x^{3/2}}dx\right.$$

$$- \exp(ct) \int_0^{M'} \frac{\sqrt{(x+M')}(1-\exp(-t(x+c)))}{(x+c)\sqrt{x}} dx \Bigg\} + \frac{G}{M'}(1-\exp(ct)) \quad (18)$$

And for $Pr = 1$

$$\tau = \sqrt{M'}\mathrm{erf}(\sqrt{M't}) + \frac{\exp(-M't)}{\sqrt{\pi t}} - \frac{G}{\pi M'} \int_0^{M'} \frac{\sqrt{(x+M')}(1-\exp(-xt))}{x^{3/2}} dx + \frac{G}{M'} \quad (19)$$

Nusselt number

From the temperature field, we now study the rate of heat transfer, which when expressed in non-dimensional form, is given by

$$Nu = -\frac{1}{\theta(0)} \frac{d\theta}{dy}\bigg|_{y=0} = +\frac{1}{\theta(0)}$$

in view of boundary condition (9).

4 Discussion

In order to understand the effects of different parameters in the problem, velocity profiles, temperature profiles, skin-friction and Nusselt number have been discussed by assigning numerical values to various parameters. Figure 1 shows the temperature profiles against η (the distance from the plate). It exhibits the effects of Prandtl number Pr and time t on temperature. It is observed that temperature increases with increasing time. The magnitude of temperature is maximum at the plate than it falls exponentially and finally tends to zero for both air (Pr = 0.71) and water (Pr = 7). Temperature is greater for air than water. It is because that of thermal conductivity of fluid decreases with increasing Pr, therefore thermal boundary layer thickness decreases with increasing Pr. Figure 2 shows the effect of Grashof number G, Prandtl number Pr and time t on velocity against η. It is evident from the figure that an increase in G, leads to an increase in velocity. It is because increasing G gives rise to buoyancy effects resulting in more induced flows. In addition, velocity decreases with increasing time for both

Pr = 0.71 and Pr = 7. The magnitude of velocity is maximum at the plate and decay to zero asymptotically. Velocity falls rapidly near the plate ($\eta \leq 1$) and slowly faded away from the plate for both air and water. Magnitude of velocity for air is greater than water. Physically, this is possible because fluids with high Prandtl number have high viscosity hence move slowly.Figure 3 depicts the effect of magnetic parameter M, permeability parameter K and Prandtl number Pr on velocity against η. It is clear from figure that magnitude of velocity is maximum at the plate, then it falls sharply near the plate ($\eta \leq 1.5$) and tends to zero asymptotically for both the cases when M is fixed or K is fixed. It is observed

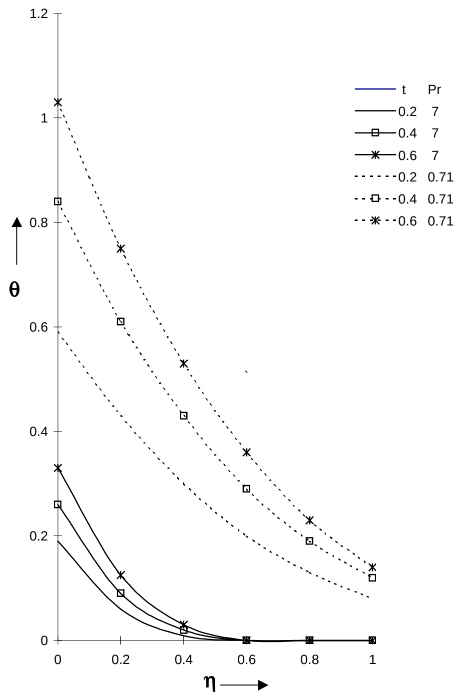

Figure 1:
Velocity profiles when $M = 2$, $K = 0.5$

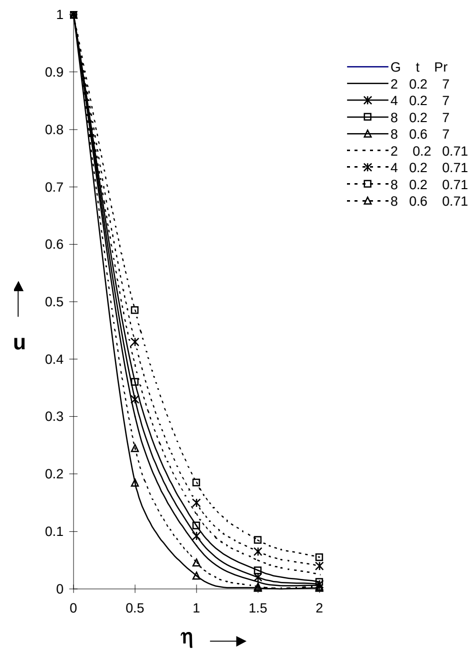

Figure 2:
Temperature profiles

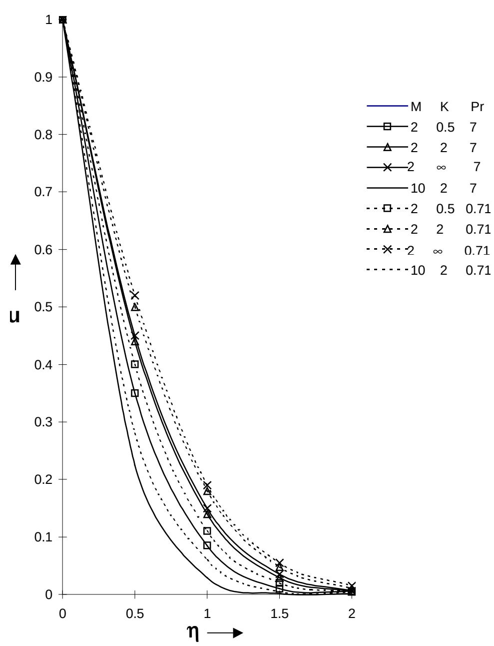

Figure 3:
Velocity profiles when $G = 4$,$t = 0.2$

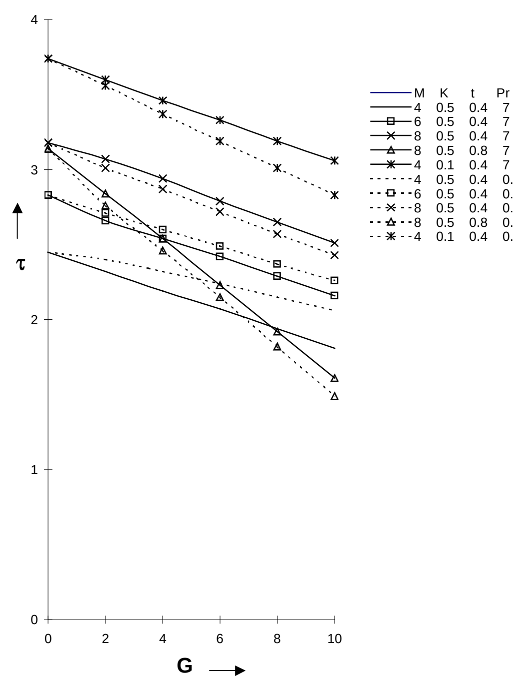

Figure 4:
Skin-friction profiles

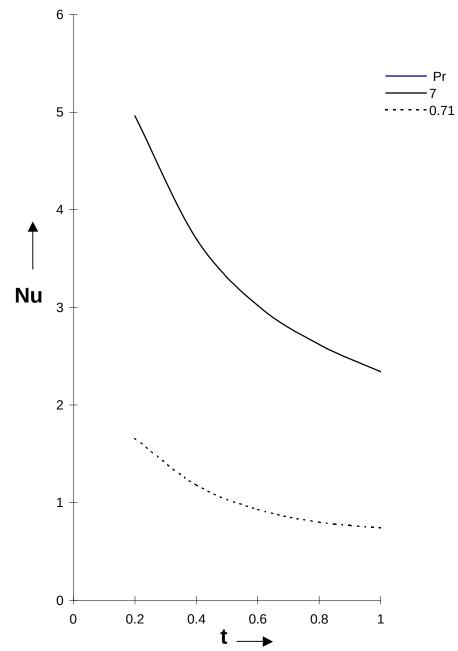

Figure 5: Nusselt number

that on increasing K, the velocity increases throughout the flow region for both Pr = 7 and Pr = 0.71. On the other hand, velocity decreases with increasing M for both air and water. It is because the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. The magnitude of velocity for air is greater than water.

The skin-friction at the plate against G- Grashof number is presented in figure 4. The results show that an increase of G, decreases the skin-friction for both air and water. Skin-friction also decreases with increasing K due to retarding effect of porous medium on the flow. Skin-friction increases with increasing M. The presence of magnetic field accelerates the flow. The magnitude of skin-friction for air is greater than water for lower values of M ($M \leq 6$) and higher values of K ($K \geq 0.5$) but reverse effect is observed for water when $M > 6$ or $K < 0.5$. When time t is increased, skin-friction falls sharply with G.Figure 5 shows coefficient of heat transfer (Nusselt number) against time t. Nusselt number decreases sharply for water and slowly for air with increasing time. Nusselt number for water is greater than air. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities and therefore heat is able to diffuse away from the plate more rapidly than for higher values of Pr. Hence the rate of heat transfer is reduced.

Nomenclature

B_0	magnetic field component along y'-axis
C_p	specific heat at constant pressure
G	Grashof number
g	acceleration of gravity
i	$\sqrt{-1}$
K'	the permeability of medium
K	the permeability parameter
M	magnetic parameter
Pr	Prandtl number
q'	the constant heat flux at the plate
T'	temperature of fluid near the plate
T'_∞	temperature of the fluid far away from the plate
t'	time in x', y' coordinate system
t	time in dimensionless co-ordinate
U_0	velocity of plate
u'	velocity component in x'-direction
u	dimensionless velocity component
x', y'	coordinate system
x, y	dimensionless coordinates

Greek symbols

β	coefficient of volume expansion
κ	thermal conductivity of the fluid
σ	electrical conductivity of the fluid
ν	kinematic viscosity
μ	viscosity
θ	non-dimensional temperature
ρ	density of fluid

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