Vol. XI, Nº 1,2 Nov. (2003)
Matemáticas: 45–55

Matemáticas:
Enseñanza Universitaria
©Escuela Regional de Matemáticas
Universidad del Valle - Colombia

Injection and suction effects on three-dimensional unsteady flow and heat transfer between two parallel porous plates

R.C. Chaudhary Bhupendra Kumar Sharma

Abstract

The problem of unsteady three-dimensional flow of an incompressible viscous fluid between two horizontal parallel porous plates with transverse sinusoidal injection of the fluid at the stationary plate and with constant suction through the plate in uniform motion has been studied. The moving plate is kept at oscillating wall temperature while the stationary plate is at constant temperature. Analytical expressions for velocity, temperature, and rate of heat transfer are obtained and discussed with the help of graphs and tables.

Keywords: Three-dimensional Unsteady flow, Heat transfer, Injection and Suction, Porous plates.

1 Introduction

The problem of laminar flow control is gaining considerable importance in the field of aeronautical engineering in view of its application to reduce drag and hence the vehicle powers requirements by a substantial amount. The increase in the drag coefficient may be prevented by the suction of the fluid and heat transfer from the boundary layer to the wall. The various theoretical and experimental studies of different arrangements and configurations of suction holes and slits have been compiled by Lachmann [1].

To reduce the drag by increasing suction alone is uneconomical, as the energy consumption of the suction pump will be more. Therefore, the method of "Cooling of the wall" in controlling the laminar flow together with the application of suction has become more useful and has consequently been focused in recent times. Most of the investigators have however confined themselves to two-dimensional flows. The exact solution of the plane Couette flow with transpiration cooling has been studied by Eckert [2]. He considered the uniform injection and suction at the porous plates and hence the problem remained two-dimensional. However, situations may arise where the flow fields may be essentially

three-dimensional. One such example is where variation in the suction or injection velocity distribution is transverse to the potential flow. Gersten and Gross [3] have studied the effect of transverse sinusoidal suction velocity distribution on flow and heat transfer over a plane wall with constant temperature. Singh [4, 5] and Singh [6] extended this idea further to horizontal and vertical porous plates with constant wall temperature. Singh [7] further studied the Couette flow with transpiration cooling to get a better record of the sinusoidal injection velocity. However, the plates are kept at constant temperature.

The problems involving a periodic surface temperature are of considerable practical importance in estimating the periodic temperatures (and periodic thermal stresses) in the walls of combustion engines. Hence, the aim of this paper is to study the effects of injection and suction on the unsteady three-dimensional flow and heat transfer, caused by the periodic injection velocity perpendicular to the flow direction at the stationary plate, while the upper moving plate is kept at periodic wall temperature.

2 Formulation of the problem

We consider the flow of a viscous incompressible fluid between two parallel flat porous plates. The upper plate in uniform motion with velocity U is subjected to a constant suction V_0 velocity and the lower to a transverse sinusoidal injection velocity distribution of the form

$$V^*(z^*) = V_0\left(1 + \varepsilon \cos\frac{\pi z^*}{d}\right) \tag{1}$$

where ε is a positive constant quantity ($\ll 1$). Without any loss of generality, the distance d between the plates is taken equal to the wave length of the injection velocity. All physical quantities are independent of x^* for this problem of fully developed laminar flow but the flow remains three-dimensional due to the injection velocity (1). Thus, under the usual Boussinesq approximation, the flow is governed by the following equations:

Continuity equation

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0. \tag{2}$$

Momentum equations

$$\rho^*\left(\frac{\partial u^*}{\partial t^*}+v^*\frac{\partial u^*}{\partial y^*}+w^*\frac{\partial u^*}{\partial z^*}\right)= \mu\left(\frac{\partial^2 u^*}{\partial y^{*2}}+\frac{\partial^2 u^*}{\partial z^{*2}}\right), \quad (3)$$

$$\rho^*\left(\frac{\partial v^*}{\partial t^*}+v^*\frac{\partial v^*}{\partial y^*}+w^*\frac{\partial v^*}{\partial z^*}\right)= -\frac{\partial p^*}{\partial y^*}+\mu\left(\frac{\partial^2 v^*}{\partial y^{*2}}+\frac{\partial^2 v^*}{\partial z^{*2}}\right), \quad (4)$$

$$\rho^*\left(\frac{\partial w^*}{\partial t^*}+v^*\frac{\partial w^*}{\partial y^*}+w^*\frac{\partial w^*}{\partial z^*}\right)= -\frac{\partial p^*}{\partial z^*}+\mu\left(\frac{\partial^2 w^*}{\partial y^{*2}}+\frac{\partial^2 w^*}{\partial z^{*2}}\right). \quad (5)$$

Energy equation:

$$\rho^* C_p\left(\frac{\partial T^*}{\partial t^*}+v^*\frac{\partial T^*}{\partial y^*}+w^*\frac{\partial T^*}{\partial z^*}\right)=K\left(\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{\partial^2 T^*}{\partial z^{*2}}\right). \quad (6)$$

The boundary conditions are

$$\begin{cases} y^*=0;\ u^*=0,\ v^*=V_0(1+\varepsilon\cos\frac{\pi z^*}{d}),\ w^*=0,\ T^*=T_0 \\ y^*=d;\ u^*=U,\ v^*=V_0,\ w^*=0,\ T^*=T_1+\varepsilon(T_1-T_0)e^{i\omega^* t^*} \end{cases} \quad (7)$$

Now introducing the following non-dimensional quantities

$$y=\frac{y^*}{d},\quad z=\frac{z^*}{d},\quad t=\frac{V_0 t^*}{4d},\quad \omega=\frac{4d\omega^*}{V_0},$$
$$u=\frac{u^*}{U},\quad v=\frac{v^*}{V_0},\quad w=\frac{w^*}{V_0},\quad \mathrm{Pr}=\frac{\mu C_p}{K},\quad p=\frac{p^*}{\rho^* V_0^2},$$
$$\theta=\frac{T^*-T_0}{T_1-T_0},\quad \lambda(\text{injection parameter})=\frac{V_0 d}{\nu}$$

we get

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \quad (8)$$

$$\frac{1}{4}\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z} = \frac{1}{\lambda}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right), \quad (9)$$

$$\frac{1}{4}\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y}+\frac{1}{\lambda}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right), \quad (10)$$

$$\frac{1}{4}\frac{\partial w}{\partial t}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z}+\frac{1}{\lambda}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right), \quad (11)$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t}+v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z} = \frac{1}{\lambda\,\mathrm{Pr}}\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right). \quad (12)$$

The corresponding boundary conditions reduce to

$$\begin{cases} y=0; \quad u=0, \quad v=1+\varepsilon\cos\pi z, \quad w=0, \quad T=0. \\ y=1; \quad u=1, \quad v=1, \quad w=0, \quad T=1+\varepsilon e^{i\omega t}. \end{cases} \quad (13)$$

3 Solution

When the amplitude of injection velocity $\varepsilon \ll 1$, we assume the solution in the neighbourhood of the plate of the form

$$f(y,z,t) = f_0(y) + \varepsilon f_1(y,z,t) + \varepsilon^2 f_2(y,z,t) + \cdots \tag{14}$$

where f stands for any of u, v, w, p and θ. When $\varepsilon = 0$ the problem is reduced to the well known two-dimensional flow with constant injection and suction at both plates. The solution of this two-dimensional problem is

$$\begin{cases} u_0(y) = \frac{e^{\lambda y}-1}{e^{\lambda}-1}, \quad v_0 = 1, \quad w_0 = 0, \quad p_0 = \text{constant} \\ \theta_0 = \frac{e^{\lambda y}-1}{e^{\lambda}-1}. \end{cases} \tag{15}$$

Taking into account the solutions of the transverse velocity components v_0 and w_0, the terms on the comparison of coefficients of ε give the following equations:

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0, \tag{16}$$

$$\frac{1}{4}\frac{\partial u_1}{\partial t} + v_1\frac{\partial u_0}{\partial y} + w_1\frac{\partial u_1}{\partial z} = \frac{1}{\lambda}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right), \tag{17}$$

$$\frac{1}{4}\frac{\partial v_1}{\partial t} + \frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{1}{\lambda}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right), \tag{18}$$

$$\frac{1}{4}\frac{\partial w_1}{\partial t} + \frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{1}{\lambda}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right), \tag{19}$$

$$\frac{1}{4}\frac{\partial \theta_1}{\partial t} + v_1\frac{\partial \theta_0}{\partial y} + \frac{\partial \theta_1}{\partial z} = \frac{1}{\lambda \Pr}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right), \tag{20}$$

with boundary conditions

$$\begin{cases} y = 0; \quad u = 0, \quad v_1 = \cos \pi z, \quad w_1 = 0, \quad \theta_1 = 0. \\ y = 1; \quad u_1 = 0, \quad v_1 = 0, \quad w_1 = 0, \quad \theta_1 = e^{i\omega t}. \end{cases} \tag{21}$$

This is the set of linear partial differential equations which describe the three-dimensional flow. In order to solve these equations we separate the variables y, z and t in the following manner:

$$u_1(y,z,t) = u_{11}(y)\, e^{i\omega t} + u_{12}(y)\cos \pi z \tag{22}$$

$$v_1(y,z,t) = v_{11}(y)\, e^{i\omega t} + v_{12}(y)\cos \pi z \tag{23}$$

$$w_1(y,z,t) = -\left[z v_{11}(y)\, e^{i\omega t} + \frac{1}{\pi} v_{12}(y)\sin \pi z\right] \tag{24}$$

$$p_1(y,z,t) = p_{11}(y)\, e^{i\omega t} + p_{12}(y)\cos \pi z \tag{25}$$

$$\theta_1(y,z,t) = \theta_{11}(y)\, e^{i\omega t} + \theta_{12}(y)\cos \pi z \tag{26}$$

Equations (23) and (24) are chosen so that the equation of continuity (16) is satisfied. Substituting equations (22) to (26) in equations (17) to (21) and equating the coefficient of harmonic and non-harmonic terms, we get the following equations:

$$u_{11}'' - \lambda u_{11}' - \frac{\lambda i\omega u_{11}}{4} = \lambda u_0' v_{11} \tag{27}$$

$$u_{12}'' - \lambda u_{12}' - \pi^2 u_{12} = \lambda u_0' v_{12} \tag{28}$$

$$\begin{cases} y = 0; & u_{11} = 0, & u_{12} = 0. \\ y = 1; & u_{11} = 0, & u_{12} = 0. \end{cases} \tag{29}$$

$$v_{11}'' - \lambda v_{11}' - \frac{\lambda i\omega v_{11}}{4} = \lambda p_{11}' \tag{30}$$

$$v_{12}'' - \lambda v_{12}' - \pi^2 v_{12} = \lambda p_{12}' \tag{31}$$

$$v_{11}''' - \lambda v_{11}'' - \frac{\lambda i\omega v_{11}'}{4} = 0 \tag{32}$$

$$v_{12}''' - \lambda v_{12}'' - \pi^2 v_{12}' = \lambda \pi^2 p_{12} \tag{33}$$

$$\begin{cases} y = 0; & v_{11} = 0, & v_{12} = 1, & v_{12}' = 0, & v_{11}' = 0. \\ y = 1; & v_{11} = 0, & v_{12} = 1, & p_{11} = 0, & p_{12} = 0. \end{cases} \tag{34}$$

$$\theta_{11}'' - \lambda \Pr \theta_{11}' - \frac{\lambda \Pr i\omega \theta_{11}}{4} = \lambda \Pr \theta_0' v_{11} \tag{35}$$

$$\theta_{12}'' - \lambda \Pr \theta_{12}' - \pi^2 \theta_{12} = \lambda \Pr \theta_0' v_{12} \tag{36}$$

$$\begin{cases} y = 0; & \theta_{11} = 0, & \theta_{12} = 0. \\ y = 1; & \theta_{11} = 0, & \theta_{12} = 0. \end{cases} \tag{37}$$

From these equations the solutions of u_1, v_1, w_1, p_1, θ_1 are obtained as

$$u_1(y,z) = \left[Le^{r_1 y} + Me^{r_2 y} + \frac{\lambda}{Ae^{\lambda} - 1} \left(\frac{A_1}{2r_1} e^{(\lambda + r_1)y} + \frac{A_2}{2r_2} e^{(\lambda + r_2)y} - \frac{A_3}{\pi} e^{(\lambda + \pi)y} + \frac{A_4}{\pi} e^{(\lambda - \pi)y} \right) \right] \cos \pi z \tag{38}$$

$$v_1(y,z) = \frac{1}{A} \left(A_1 e^{r_1 y} + A_2 e^{r_2 y} - A_3 e^{\pi y} - A_4 e^{-\pi y} \right) \cos \pi z \tag{39}$$

$$w_1(y,z) = \frac{1}{\pi A} (A_1 r_1 e^{r_1 y} + A_2 r_2 e^{r_2 y} - \pi A_3 e^{\pi y} + \pi A_4 e^{-\pi y}) \sin \pi z \tag{40}$$

$$p_1(y,z) = \frac{1}{A}(A_3 e^{\pi y} + A_4 e^{-\pi y})\cos \pi z \quad (41)$$

$$\theta_1(y,z,t) = \left(\frac{e^{m_1 y} - e^{m_2 y}}{e^{m_1} - e^{m_2}}\right) e^{i\omega t} + \left[\frac{\lambda \,\mathrm{Pr}^2}{A(e^{\lambda\,\mathrm{Pr}} - 1)}\left(\frac{A_1 e^{(r_1+\lambda\,\mathrm{Pr})y}}{r_1(\mathrm{Pr}+1)} + \frac{A_2 e^{(r_2+\lambda\,\mathrm{Pr})y}}{r_2(\mathrm{Pr}+1)} - \frac{A_3 e^{(\pi+\lambda\,\mathrm{Pr})y}}{\pi\,\mathrm{Pr}} + \frac{A_4 e^{(\lambda\,\mathrm{Pr}-\pi)y}}{\pi\,\mathrm{Pr}}\right) + R\, e^{s_1 y} + S e^{s_2 y}\right]\cos \pi z \quad (42)$$

where

$$\begin{aligned}
A &= 2(r_2 - r_1)\left(1 + e^{r_1+r_2}\right) - \left[(r_2 - r_1) + 2\pi\right]\left(e^{r_1+\pi} + e^{r_2-\pi}\right) \\
&\quad - \left[(r_2 - r_1) - 2\pi\right]\left(e^{r_1-\pi} + e^{r_2+\pi}\right) \\
A_1 &= (\pi - r_2)\, e^{r_2+\pi} - (\pi + r_2)\, e^{r_2-\pi} + 2r_2 \\
A_2 &= (\pi + r_1)\, e^{r_1-\pi} - (\pi - r_1)\, e^{r_1+\pi} - 2r_1 \\
A_3 &= (r_1 - r_2)\, e^{r_1+r_2} + (r_2 - \pi)\, e^{r_1-\pi} - (r_1 - \pi)\, e^{r_2-\pi} \\
A_4 &= (r_1 - r_2)\, e^{r_1+r_2} + (r_2 + \pi)\, e^{r_1+\pi} - (r_1 + \pi)\, e^{r_2+\pi} \\
r_1 &= \frac{1}{2}\left(\lambda + \sqrt{\lambda^2 + 4\pi^2}\right), \qquad r_2 = \frac{1}{2}\left(\lambda - \sqrt{\lambda^2 + 4\pi^2}\right)
\end{aligned}$$

$$L = \frac{\lambda}{A(e^{\lambda} - 1)(e^{r_1} - e^{r_2})}\left[\frac{A_1}{2r_1}\left(e^{r_2} - e^{\lambda + r_1}\right) + \frac{A_2}{2r_2}\left(e^{r_2} - e^{\lambda + r_2}\right) - \frac{A_3}{\pi}\left(e^{r_2} - e^{\lambda+\pi}\right) + \frac{A_4}{\pi}\left(e^{r_2} - e^{\lambda-\pi}\right)\right]$$

$$M = \frac{\lambda}{A(e^{\lambda} - 1)(e^{r_1} - e^{r_2})}\left[\frac{A_1}{2r_1}\left(e^{\lambda + r_1} - e^{r_1}\right) + \frac{A_2}{2r_2}\left(e^{\lambda + r_2} - e^{r_1}\right) - \frac{A_3}{\pi}\left(e^{\lambda+\pi} - e^{r_1}\right) + \frac{A_4}{\pi}\left(e^{\lambda-\pi} - e^{r_1}\right)\right]$$

$$R = \frac{\lambda\,\mathrm{Pr}^2}{A(e^{\lambda\,\mathrm{Pr}} - 1)(e^{s_1} - e^{s_2})}\left[\frac{A_1}{r_1(\mathrm{Pr}+1)}\left(e^{s_2} - e^{\lambda\,\mathrm{Pr} + r_1}\right) + \frac{A_2}{r_2(\mathrm{Pr}+1)}\left(e^{s_2} - e^{\lambda\,\mathrm{Pr} + r_2}\right) - \frac{A_3}{\pi}\left(e^{s_2} - e^{\lambda\,\mathrm{Pr}+\pi}\right) + \frac{A_4}{\pi}\left(e^{s_2} - e^{\lambda\,\mathrm{Pr}-\pi}\right)\right]$$

$$S = \frac{\lambda \,\mathrm{Pr}^2}{A(e^{\lambda\,\mathrm{Pr}}-1)(e^{s_1}-e^{s_2})}\Bigg[\frac{A_1}{r_1(\mathrm{Pr}+1)}\left(e^{\lambda\,\mathrm{Pr}+r_1}-e^{s_1}\right) + \frac{A_2}{r_2(\mathrm{Pr}+1)}\left(e^{\lambda\,\mathrm{Pr}+r_2}-e^{s_1}\right) - \frac{A_3}{\pi}\left(e^{\lambda\,\mathrm{Pr}+\pi}-e^{s_1}\right)+\frac{A_4}{\pi}\left(e^{\lambda\,\mathrm{Pr}-\pi}-e^{s_1}\right)\Bigg]$$

$$\begin{aligned} s_1 &= \frac{1}{2}\left(\lambda\,\mathrm{Pr}+\sqrt{(\lambda\,\mathrm{Pr})^2+4\pi^2}\right), \quad s_2 = \frac{1}{2}\left(\lambda\,\mathrm{Pr}-\sqrt{(\lambda\,\mathrm{Pr})^2+4\pi^2}\right) \\ m_1 &= \frac{1}{2}\left(\lambda\,\mathrm{Pr}+\sqrt{(\lambda\,\mathrm{Pr})^2+\lambda\,\mathrm{Pr}\,i\omega}\right) \\ m_2 &= \frac{1}{2}\left(\lambda\,\mathrm{Pr}-\sqrt{(\lambda\,\mathrm{Pr})^2+\lambda\,\mathrm{Pr}\,i\omega}\right). \end{aligned}$$

Substituting equations (15), (42) in equation (14), we get the expression for the temperature profiles. The temperature can now be expressed in terms of fluctuating parts as

$$\theta\,(y,z,t) = \theta_0\,(y) + \varepsilon\left[(T_r\cos\omega t - T_i\sin\omega t) + \theta_{12}\,(y)\cos\pi z\right] \qquad (43)$$

where

$$T_r + iT_i = \frac{e^{m_1 y}-e^{m_2 y}}{e^{m_1}-e^{m_2}}.$$

For $\omega t = \frac{\pi}{2}$ we can now obtain the temperature profiles as

$$\theta\left(y,z,\frac{\pi}{2\omega}\right) = \theta_0\,(y) + \varepsilon\left(\theta_{12}\,(y)\cos\pi z - T_i\right). \qquad (44)$$

4 Discussion

Fig.1 gives the main flow velocity profiles for $z = 0$ and $\varepsilon = 0.2$. If there is neither injection nor suction, the dotted line represents the well known Couette flow. This figure reveals that the velocity decreases exponentially when there is injection. For higher rate of injection the decay is greater. The maximum and minimum values of the velocities occur on the plates, which are the velocities of the plates. The secondary flow component w_1, which is due to the transverse sinusoidal injection velocity, is shown in Fig.2, for several injection parameters. The maximum of the velocity occurs in the fluid not far from the stationary plate. The transient temperature profiles are shown in Fig.3 for $z = 0$. In the region between the plates the fluid will be heated to a temperature below that of the moving plate, which is kept at maximum temperature, hence maximum

Figure 1: Main flow velocity profiles for $z = 0$ and $\epsilon = 0.2$

Figure 2: Cross flow velocity for $z = 0.5$

Figure 3: Transient temperature profiles for $\omega t = \pi/2 \quad (\epsilon = 0.2 \quad z = 0)$

Figure 4: Sinusoidal rate of heat transfer for $\epsilon = 0.2$

occurs at that plate. When the rate of injection increases, the cooling occurs in the region. It is found that the temperature in the case of air is more than that of water. For $\mathrm{Pr} = 7$ (water), the temperature decreases exponentially from the plate kept at higher temperature. For higher λ, this decay is greater. The maximum and minimum values of temperature occur on the plates.

From the temperature field we can calculate the rate of heat transfer in non-dimensional form as

$$-q = \frac{dq^*}{\kappa\,(T_1 - T_0)} = \left(\frac{\partial\theta}{\partial y}\right)_{y=0} = \left(\frac{\partial\theta_0}{\partial y}\right)_{y=0} + \varepsilon\left(\frac{\partial\theta_1}{\partial y}\right)_{y=0} \tag{45}$$

and in terms of the amplitude and the phase q, can be expressed as

$$q = q_1 + \varepsilon\,|N|\cos\left(\omega t + ga\right) \tag{46}$$

where the sinusoidal rate of heat transfer

$$q_1 = \frac{\lambda\,\mathrm{Pr}}{e^{\lambda\,\mathrm{Pr}} - 1} + \varepsilon\Bigg[Rs_1 + Ss_2 + \frac{\lambda\,\mathrm{Pr}^2}{A(e^{\lambda\,\mathrm{Pr}} - 1)}\Bigg(\frac{A_1(\lambda\,\mathrm{Pr} + r_1)}{r_1(\mathrm{Pr}+1)} + \frac{A_2(\lambda\,\mathrm{Pr} + r_2)}{r_2(\mathrm{Pr}+1)} - \frac{A_3(\lambda\,\mathrm{Pr} + \pi)}{\pi\,\mathrm{Pr}} + \frac{A_4(\lambda\,\mathrm{Pr} - \pi)}{\pi\,\mathrm{Pr}}\Bigg)\Bigg]\cos\pi z,$$

$$N_r + iN_i \;=\; \frac{m_1 - m_2}{e^{m_1} - e^{m_2}} \quad \text{and} \quad \tan\alpha = \frac{N_i}{N_r}.$$

The sinusoidal rate of heat transfer is, as Fig.4 also shows, a function of the Prandtl number Pr and injection parameter λ. When $\mathrm{Pr} = 0.71$ (air), which means the viscosity is small but the thermal conductivity is finite, the heat transfer is great. However, when the viscosity is large in comparison to the thermal diffusivity ($\mathrm{Pr} = 7$, in the case of water), the heat transfer reduces. The deviation from the numerically calculated heat transfer amounts significantly for $0.71 \leq \mathrm{Pr} \leq 7$. It is also observed from the figure that in the case of blowing (injection) the heat transfer decreases as Prandtl number increases.

Table 1

λ	$\mathrm{Pr}\,/\omega$	$\lvert N\rvert$			$\tan\alpha$		
		5	10	20	5	10	20
0.5	0.71	0.9887	0.9850	0.9299	0.8389	0.8967	0.9319
0.5	7	0.7738	0.5733	0.3377	0.9313	0.9117	0.8984

Table 2

λ	Pr $/\omega$	$\lvert N\rvert$			$\tan\alpha$		
		5	10	20	5	10	20
0.5	0.71	0.6908	0.6822	0.6165	0.7299	0.8330	0.8840
0.5	7	0.4455	0.2586	0.1078	0.9736	0.8892	0.8582

Table 1 and Table 2 give the amplitude and phase shift ($\tan\alpha$) of the rate of heat transfer for $\lambda = 0.5$ and $\lambda = 1.0$ respectively. We observe that due to the high frequency (ω) of the oscillations in temperature, the magnitude of rate of heat transfer reduces. It is further noted that the values of $|N|$ are less in water than in air. The values of $\tan\alpha$ show that there is always a phase lead in the rate of heat transfer coefficient.

Acknowledgement. One of the authors, Bhupendra Kumar Sharma, is grateful to the Council of Scientific and Industrial Research, New Delhi (India), for the award of a Junior Research Fellowship.

References

[1] G.V.Lachmann, *Boundary layer and flow control. Its principles and Application*, Vol.I II, Pergamon press, Oxford (1961).

[2] E.R.Eckert, *Heat and Mass Transfer.* McGraw Hill, New York, (1958).

[3] K.Gersten and J.F.Gross, *J. Appl. Maths. Phys.* (ZAMP), 25, 399-408, 1974.

[4] P.Singh, V.P.Sharma and U.N.Mishra, *Appl. Sci. Res., 34, 105-115 (1978).*

[5] Ibid, *Int. J. Heat Mass Transfer,* 21, 1117-1123, (1978).

[6] K.D.Singh, *J. Appl. Math. Mech.* (ZAMM), 73, 58-61, (1993).

[7] Ibid, *J. Appl. Math. Phys.* (ZAMP), 50, 661- 668, (1999).

Dirección de los autores: R.C. Chaudhary — Bhupendra Kumar Sharma Department of Mathematics, University of Rajasthan, JAIPUR–302004, India.