MATEMÁTICAS

ON THE EFFECT OF EARTHQUAKES OF SHORT DURATION ON CURVILINEAR SHAPED DAMS USING A GENERALISED SCHWARZ-CHRISTOFFEL THEORY

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Abstract

The works of Chawng, J. Fluid. Mech., Vol. 87, 343-348 (1978) and Liu, J. Fluid Mech., Vol. 165, 131-145 (1986) on the effect of earthquakes on constant slope dams are extended to the case where the surfaces of the dams have curvilinear shapes. A generalised Schwarz-Christoffel theory is used to obtain general expressions for the hydrodynamic pressure distributions on the curvilinear surfaces of the dams in terms of certain functions which define the curvilinear nature of the surfaces. Two types of dam systems are examined.

1 Introduction

It is important when designing dams that one is able to predict the earthquake-induced hydrodynamic pressure on the upstream face in case such an event actually occurs. Chwang [1], Chwang and Housner [2] and Liu [3] discussed and obtained expressions for this quan-

tity although the first to obtain an expression for the hydrodynamic pressure on certain dams, due to horizontal harmonic motions, was Westergaard [4].

Chwang [1] considered dams with infinite cross-sections using conformal mapping. In [2], the momentum-balance principle, first introduced by von Karman [5], was used in relation to a dam of infinite cross-section. In [3] Liu, also using conformal mapping, treated two-dimensional dams of finite cross-section with sides of constant slopes.

In [3] Liu claims that to investigate a dam of more general shape than the constant slope shapes he considered, one has to resort to numerical means. In the present paper we consider curvilinear shaped dams and using a generalized Schwarz-Christoffel approach we extend the above works. We derive an analytical expression for the corresponding hydrodynamic pressure due to earthquakes on two-dimensional dams of curvilinear shapes. The generalized Schwarz-Christoffel approach is that which the present author has used to treat Electrostatic(Magnetostatic) problems [6] and Hydrodynamic problems[7]. Both vertical and horizontal motions are considered, and as in the other works, we assume that the duration of the acceleration of the dam is so short and non-impulsive that we may neglect the compressibility of the water and hence the effect of surface perturbation. The author was unable to find any references on dams of curved shapes with which to compare the results of this paper.

Section 2 is devoted to the formulation of the problem while in Section 3 we derive the general formula for the mapping of the upper-half plane onto the interior of the finite cross-sectional dam with the real axis going onto the boundary of the dam. From this formula we obtain mapping formulae corresponding to special cases, for example, where the cross-section is infinite with one side curvilinear or where the cross-section is finite and one side is curvilinear and the other of constant slope. The formal solution to the general shaped dam is contained in Section 4. In Section 5, we show how to derive certain functions which determine the curvilinear nature. Section 6 contains the explicit solution for certain general problems, and Section 7 is devoted to two sets of applications.

2 Formulation of the problem

In Fig.1 we show the cross-section of the dam and reservoir system. We shall take the origin of coordinates, as in [3], to be at B so that the free-surface is $y = D$. Due to the earthquake, the whole system is assumed to undergo a constant acceleration $\underline{a}$ which makes an angle γ with the x-axis. We shall assume that the liquid is incompressible and invicid so that the hydrodynamic pressure satisfies

$$\nabla^2 p = 0$$

We shall also assume that the acceleration acts for only a very short time so that the free-surface may be considered as unaffected. We thus have the condition

$$p = 0 \quad \text{at} \quad y = D \tag{1}$$

as the only free-surface condition. The other conditions to be satisfied are

(i)

$$\frac{\partial p}{\partial n_1} = -\rho \underline{a} \cdot \underline{n}_1 \qquad \text{on the dam face AB} \tag{2}$$

(ii)

$$\frac{\partial p}{\partial n_2} = -\rho \underline{a} \cdot \underline{n}_2 \qquad \text{on the dam face BC} \tag{3}$$

where $\underline{n}_1$ and $\underline{n}_2$ are the outward normals along AB and BC respectively and the acceleration $\underline{a}$ is given by $\underline{a} = (a_0 \cos\gamma,\ a_0 \sin\gamma)$. We note that the right-hand sides of eqns.(2) and (3) are prescribed.

3 Derivation of the transformation formulae

The cross-section of the general shaped dam is shown in Fig.1 by the curvilinear lines AB and BC.
AC is the unaffected water surface. The dashed lines AB, BC will be used to derive the formula. Let the angles $C\hat{A}B$, $A\hat{B}C$, $B\hat{C}A$ be α, δ, ϕ respectively where $\alpha+\delta+\phi = \pi$. (Our notation is similar to that of Liu with $\phi = \beta$.)

The Schwarz-Christoffel transformation which maps the upper-half t - plane with $t = 0$ and $t = 1$, corresponding to A and B respectively, is

$$\frac{dz}{dt} = Ke^{i\phi}t^{\frac{\alpha}{\pi}-1}(t-1)^{\frac{\sigma}{\pi}-1},\ K > 0\ .$$

Following the technique used in [6] and [7] and inserting the appropriate shape factors, $\exp\left(\frac{1}{\pi}\int_0^1 \frac{\beta_0(u)du}{u-t}\right)$ and $\exp\left(\frac{1}{\pi}\int_1^\infty \frac{\beta_+(u)du}{u-t}\right)$, to produce curved sections AB and BC respectively, we obtain

$$\frac{dz}{dt} = Ke^{i\phi}t^{\frac{\alpha}{\pi}-1}(t-1)^{\frac{\delta}{\pi}-1}S(t) \tag{4}$$

where $K > 0$, $S(t) = S_1(t)S_2(t)$, $S_1(t) = \exp\left(\frac{1}{\pi}\int_0^1 \frac{\beta_0(u)du}{u-t}\right)$ and $S_2(t) = \exp\left(\frac{1}{\pi}\int_1^\infty \frac{\beta_+(u)du}{u-1}\right)$.
This formula can be shown to be independent of the original angles α and δ.
We note that if we put $\beta_0 \equiv 0 \equiv \beta_+$ we obtain the formula for the triangular dam of [3]. From eqn.(4) we obtain the following:

a) **Dam of infinite cross-section with a curved upstream face**

To obtain the transformation formula for the mapping of a dam of infinite cross-section and curved upstream face we take $\phi \equiv 0 \equiv \beta_+(t)$ giving

$$\frac{dz}{dt} = Kt^{\frac{\alpha}{\pi}-1}(t-1)^{\frac{\delta}{\pi}-1}S_1(t)\ . \tag{5}$$

This extends eqn.(2) of [1]. The cross-section to which it corresponds is shown in Fig.2

b) **Dam of finite cross-section with a straight bottom**

If we set $\beta_+(t) \equiv 0$ in eqn.(4) we get

$$\frac{dz}{dt} = Ke^{i\phi}t^{\frac{\alpha}{\pi}-1}(t-1)^{\frac{\delta}{\pi}-1}S_1(t) \tag{6}$$

This formula corresponds to the shape in Fig.3 where we have a finite cross-section with curved upstream and constant sloped bottom.
It is clear that we can obtain other dam shapes by properly choosing β_0, β_+. Eqn.(4) gives the formula for the general system which has both up-stream and down-stream faces curvilinear.

4 Formal solution for the general shaped dam

The equations of the surfaces AB and BC are given in terms of a parameter, t, say $x = x(t)$, $y = y(t)$. For such surfaces the outward unit normals are given by

$$\underline{n} = \frac{\dot{y}_d \underline{i} - \dot{x}_d \underline{j}}{\sqrt{\dot{x}_d^2 + \dot{y}_d^2}} \quad \text{and} \quad \underline{n} = \frac{\dot{y}_b \underline{i} - \dot{x}_b \underline{j}}{\sqrt{\dot{x}_b^2 + \dot{y}_b^2}} \tag{7}$$

respectively. The subscripts d and b are used as in [1] to refer to the dam's surface and reservoir's bottom. Using eqn.(7) the boundary conditions on the solid surfaces, (i.e. eqns.(2) and (3)),become, respectively,

$$\frac{\partial\, p}{\partial\, n_d} = \frac{\rho a_o(-\dot{y}_d \cos\gamma + \dot{x}_d \sin\gamma)}{\sqrt{\dot{x}_d^2 + \dot{y}_d^2}} \tag{8}$$

and

$$\frac{\partial\, p}{\partial\, n_b} = \frac{\rho a_0(-\dot{y}_b \cos\gamma + \dot{x}_b \sin\gamma)}{\sqrt{\dot{x}_b^2 + \dot{y}_b^2}} \tag{9}$$

where $\underline{a} = (a_0 \cos\gamma,\ a_0 \sin\gamma)$.
We now introduce the analytic function $W(z) = p + iq$ where $z = x + iy$ and integrating the Cauchy-Riemann equation $\dfrac{\partial\, p}{\partial\, n} = \dfrac{\partial\, q}{\partial\, s}$ we obtain from eqns.(8) and (9)

$$q = \rho a_0(x_d \sin\gamma - y_d \cos\gamma) \tag{10}$$

on AB, and on BC

$$q = \rho a_0(x_b \sin\gamma - y_b \cos\gamma) \tag{11}$$

(In deriving eqns.(10) (11) we used $\frac{ds_{d,b}}{dt} = \sqrt{\dot{x}_{d,b}^2 + \dot{y}_{d,b}^2}$.)
In the t-plane, where $t = \xi + i\eta$, along its real axis we have,

$$p = Re\{W\} = 0 \quad -\infty < \xi < 0,$$

and

$$q = Im\{W\} = \begin{cases} \rho a_0(x_d \sin\gamma - y_d \cos\gamma), & 0 < \xi < 1 \\ \rho a_0(x_b \sin\gamma - y_b \cos\gamma), & \xi > 1. \end{cases} \tag{12}$$

As in [2] we note that we have mixed boundary conditions for $W(t)$ and defining the function

$$H(t) = t^{-\frac{1}{2}} W(t) \tag{13}$$

where we take the positive branch of the square root function, we get

$$H = \begin{cases} 0 & t < 0 \\ t^{-\frac{1}{2}}\ \rho\ a_0(x_d \sin\gamma - y_d \cos\gamma) & 0 < t < 1 \\ t^{-\frac{1}{2}}\ \rho\ a_0(x_b \sin\gamma - y_b \cos\gamma) & t > 1 \end{cases}$$

where (x_d, y_d) and (x_b, y_b) are the coordinates of a point on the up-stream and down-stream surfaces respectively as mentioned above. We can now write H as

$$H(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\mathrm{Im}\{H(u)\}\, du}{u - t}$$

by the Riemann-Hilbert theorem. Explicitly, we have for W

$$\begin{aligned} W(t) \;=\; & \frac{\rho\ a_0}{\pi} t^{\frac{1}{2}} \left(\int_0^1 \frac{u^{-\frac{1}{2}}(x_d \sin\gamma - y_d \cos\gamma)}{u - t} du \right. \\ & \left. + \int_1^{\infty} \frac{u^{-\frac{1}{2}}(x_b \sin\gamma - y_b \cos\gamma) du}{u - t} \right) \end{aligned} \tag{14}$$

Integrating by parts and taking the real part of W we can express $p(t)$, for $0 < t < 1$, after some simplification, by

$$p(t) =$$

$$\frac{K\ \rho\ a_0}{\pi}\int_0^1 \ln\left|\frac{u^{\frac{1}{2}}+t^{\frac{1}{2}}}{u^{\frac{1}{2}}-t^{\frac{1}{2}}}\right| u^{\frac{\alpha}{\pi}-1}\ (1-u)^{\frac{\delta}{\pi}-1}\ e^{R_3(u)}\sin(\gamma+\alpha-\beta_0(u))du$$

$$-\frac{K\ \rho\ a_0}{\pi}\int_1^\infty \ln\left|\frac{u^{\frac{1}{2}}+t^{\frac{1}{2}}}{u^{\frac{1}{2}}-t^{\frac{1}{2}}}\right| u^{\frac{\alpha}{\pi}-1}(u-1)^{\frac{\delta}{\pi}-1}e^{R_4(u)}\sin(\phi-\gamma+\beta_+(u))du \tag{15}$$

This is an exact expression for the hydrodynamic pressure on the curvilinear surfaces of the dam of general shape. It is easily verified that we may reproduce eqn.(25) of [3] and eqn.(11a) of [1] for the same quantity. Now noting that the dependence of on t in eqn.(15) is the same as eqn.(25) in [3] we differentiate $p(\xi)$ with respect to ξ to get

$$\frac{dp(\xi)}{d\xi} =$$

$$\frac{K\rho a_0}{\pi\sqrt{\xi}}\left\{\int_0^1 u^{\frac{\alpha}{\pi}-1}(1-u)^{\frac{\delta}{\pi}-1}\frac{u^{\frac{1}{2}}}{u-\xi}e^{R_3(u)}\sin(\gamma+\alpha-\beta_0(u)du\right.$$
$$\left.-\int_1^\infty \frac{u^{\frac{\alpha}{\pi}-1}}{u-\xi}(1-u)^{\frac{\delta}{\pi}-1}e^{R_4(u)}\sin(\phi-\gamma+\beta_+(u))du\right\}$$

Since we shall only discuss the shapes with $\beta_+(u)\equiv 0$, we have

$$\frac{dp(\xi)}{d\xi} =$$

$$\frac{K\rho a_0}{\pi\sqrt{\xi}}\left\{\int_0^1 u^{\frac{\alpha}{\pi}-1}(1-u)^{\frac{\delta}{\pi}-1}\frac{u^{\frac{1}{2}}}{u-\xi}e^{R_3(u)}\sin(\gamma+\alpha-\beta_0(u))du\right.$$
$$\left.-\sin(\phi-\gamma)\int_1^\infty \frac{u^{\frac{\alpha}{\pi}-1}}{u-\xi}(u-1)^{\frac{\delta}{\pi}-1}e^{R_4(u)}du\right\} \tag{16}$$

The exponential factors $e^{R_3(u)}, e^{R_4(u)}$ and the term with $\beta_0(u)$ in the first integral give the deviation from previous works.
We cannot go any further until we know $R_3(u)$ and $R_4(u)$, and these can be found only after $\beta_0(u)$ is chosen. This function will now be obtained.

5 Determination of the curvature function.

From eqn.(4) we have $\arg\left(\frac{dz}{dt}\right) = -\alpha + \beta_0$ for $0 < t < 1$. At $A(t = 0)$ we have from Fig.1 $\arg\left(\frac{dz}{dt}\right) = -\theta_0$ and so we take $\beta_0(0) = \alpha - \theta_0$. This means that the angle which the tangent to the dam's surface at A makes with the unperturbed water level is θ_0. At $B(t = 1)$ we take $\arg\left(\frac{dz}{dt}\right) = -\theta_1$ and hence $\beta_1(0) = \alpha - \theta_1$. This implies that the angle which the tangent to the dam's surface at $B(t = 1)$ makes with the positive real axis is $-\theta_1$. Noting also that $\beta_0(t)$ must be a decreasing function in (0,1) we choose

$$\begin{aligned} &\beta_0(t) = \alpha - \theta_0 - (\theta_1 - \theta_0)t^N, \\ &= \gamma_0 - \gamma_1 t^N,\ \gamma_0 = \alpha - \theta_0,\ \gamma_1 = \theta_1 - \theta_0\ \gamma_0 > 0,\ \gamma_1 > 0 \end{aligned} \tag{17}$$

where $N = 1, 2, 3 \cdots$ This polynomial expression for $\beta_0(t)$ is used because it is easy to integrate analytically. We also note that if $\theta_1 = \theta_0 = \alpha$ then $\beta_0(t) \equiv 0$ and we reproduce the constant slope dam expressions.

6 Explicit solution of certain general problems

In the general problem discussed above, the functions R_3 and R_4 will have the following forms:
With $\beta_0(u) = \gamma_0 - \gamma_1 t^N$, $\beta_+(u) \equiv 0$, R_3 and R_4 are given by

$$R_3(u) = h(u) + \frac{\beta_0(u)}{\pi} \ln\left(\frac{1-u}{u}\right),\ 0 < u < 1,$$

$$R_4(u) = h(u) + \frac{\beta_0(u)}{\pi} \ln\left(\frac{u-1}{u}\right),\ u > 1$$

where

$$h(u) = \frac{-\gamma_1}{\pi}\left(\frac{1}{N} + \frac{u}{N-1} + \frac{u^2}{N-2} + \cdots\cdots\cdots + \frac{u^{N-2}}{2} + \frac{u^{N-1}}{1}\right).$$

Returning to eqn.(16), following Liu we evaluate the integrals by contour integration. For the case with $\beta_0(t) = \gamma_0 - \gamma_1 t^N$, we get for the integrals in eqn.(16) after contour integrating and some algebra, that

$$\int_0^1 u^{\frac{\alpha}{\pi}-1}(1-u)^{\frac{\delta}{\pi}-1}\frac{u^{\frac{1}{2}}}{u-\xi}e^{R_3(u)}\sin(\gamma+\alpha-\beta_0(u))du$$
$$= A_{0\infty}(\xi)\cos\gamma - C(\xi)\cos(\gamma+\theta_0+\gamma_1\xi) - \sin(\gamma-\phi)A_{1\infty}$$

hence from eqn.(16)

$$\frac{dp}{d\xi} = \left(\frac{K\rho a_0}{\pi\sqrt{\xi}}\right)(A_{0\infty}(\xi)\cos\gamma - C(\xi)\cos(\gamma+\theta_0+\gamma_1\xi)) \qquad (18)$$

where

$$A_{0\infty}(\xi) = \int_0^\infty \frac{u^{\frac{\alpha}{\pi}-1}(1+u)^{\frac{\delta}{\pi}-1}u^{\frac{1}{2}}}{u+\xi}\exp[h(-u)+\frac{\beta_0(-u)}{\pi}\ln\left(\frac{1+u}{u}\right)]du,$$

$$A_{1\infty}(\xi) = \int_1^\infty \frac{u^{\frac{\alpha}{\pi}-1}(u-1)^{\frac{\delta}{\pi}-1}u^{\frac{1}{2}}}{u-\xi}\exp[h(u)+\frac{\beta_0(u)}{\pi}\ln\left(\frac{u-1}{u}\right)]du,$$

$$C(\xi) = \pi\xi^{\frac{\alpha}{\pi}-1}(1-\xi)^{\frac{\delta}{\pi}-1}\xi^{\frac{1}{2}}\exp[h(\xi)+\frac{\beta_0(\xi)}{\pi}\ln\left(\frac{1-\xi}{\xi}\right)].$$

Finally integrating eqn.(18) following Liu[3] we get, for $N=1$,

$$\frac{p(\xi)}{\rho a_0 h} = \frac{2\cos\gamma}{\pi\lambda_2}\int_0^\infty e^{\frac{-\gamma_1}{\pi}}u^{\frac{\theta_0}{\pi}-1-\frac{u\gamma_1}{\pi}}(1+u)^{\frac{-(\phi+\theta_0)}{\pi}+\frac{u\gamma_1}{\pi}}\tan^{-1}\left(\frac{\xi}{u}\right)^{\frac{1}{2}}du$$

$$-\frac{1}{\lambda_2}\int_0^\xi e^{\frac{-\gamma_1}{\pi}}u^{\frac{\theta_0}{\pi}-1+\frac{u\gamma_1}{\pi}}(1-u)^{\frac{-(\phi+\theta_0)}{\pi}+\frac{\gamma_1(1-u)}{\pi}}\cos(\gamma+\theta_0+u\gamma_1)du \qquad (19)$$

where $k=\frac{D}{\lambda_2}$, $\gamma_1=\theta_1-\theta_0$ and

$$\lambda_2 = \int_0^1 e^{\frac{-\gamma_1}{\pi}}u^{\frac{\theta_0}{\pi}-1+\frac{u\gamma_1}{\pi}}(1-u)^{\frac{-(\phi+\theta_1)}{\pi}+\frac{\gamma_1(1-u)}{\pi}}\sin(\theta_0+u\gamma_1)du.$$

We easily verify that if $\theta_0=\theta_1$ and $\alpha=\theta_0$ i.e. $\beta_0\equiv 0$, we reproduce the results of Liu.

7 Applications

Following Liu we consider only horizontal ground accelerations ($\gamma = 0$) and $N = 1$ in eqn.(17).

a) Horizontal ground accelerations ($\gamma = 0$) on a semi-infinite cross-sectional dam($\phi = 0$).

Using eqn.(19), $\frac{p(\xi)}{Da_0\rho}$, is plotted against the vertical dimensionless distance y/D in Fig.4 for values of θ_0, θ_1 as indicated.

b) Horizontal ground accelerations ($\gamma = 0$) on a finite cross-sectional dam($\phi \neq 0$).

In this case $\frac{p(\xi)}{Da_0\rho}$ against y/D is shown in Figs.(5), (6), (7), (8) for the values of $\theta_0, \theta_1\phi$ as indicated. We can easily reproduce Liu's graphs by putting $\beta_0(t) \equiv 0$, however we shall not draw them here.

8 Concluding remarks

Using a generalised Schwarz-Christoffel approach, works of Chwang [1] and Liu [3] on the effects of earthquakes on dams have been extended to accommodate curvilinear shaped surfaces of these structures. We derived an analytical expression for the hydrodynamic pressure on the curved upstream surface of the dam. This is similar to that derived by Liu and the effect of curving the dam's surface is easily seen. The new expression for the pressure is dependent on a function, $\beta(t)$, which was introduced to produce the curving of the surfaces. On setting $\beta(t) \equiv 0$ we can reproduce the results of Chwang[1] and Liu[3].

The dimensionless hydrodynamic pressure, $\frac{p}{\rho a_0 D}$, was plotted against the dimensionless vertical height $\frac{y}{D}$ for different angles θ_0, θ_1 where $-\theta_0$ is the angle which the upper end of the upstream surface makes with the flat water-surface, while $-\theta_1$ is the angle which the toe of the upstream surface makes with the straight bottom which was inclined at an angle ϕ to the horizontal.

For dams with a semi-infinite cross-section, i.e. $\phi = 0$, the pressure was a maximum when $\theta_1 = \frac{\pi}{2}$ whatever the value of θ_0. For other values of $\theta_1 < \frac{\pi}{2}$ the maximum occurred at a point above the base. This was similar to Liu's results.

For small values of θ_0 there was a point of inflection in the pressure distribution near the centre of the upstream surface.

For dams with bottom inclination $\phi \neq 0$, we found again that when the toe angle $\theta_1 = \frac{\pi}{2}$ the pressure distribution was a maximum at the toe and for other toe angles the maximum occurred at a point above the toe. For some of these dams with $\phi \neq 0$ we also found that for small θ_0 angles the pressure became negative near the toe of the dam suggesting cavitation could occur.

As mentioned in the introduction the author was unable to find any other work on curvilinear shaped dams with which these results could be compared.

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Figure captions

Fig.1 Cross-section of a general shaped dam

Fig.2 Dam of semi-infinite cross-section and curved upstream surface

Fig.3 Dam of finite cross-section with plane downstream face

Fig.4 Pressure distributions on upstream surfaces of dams of semi-infinite cross-sections

Fig.5 Pressure distributions on upstream surfaces of dams with a bottom inclination of 15°

Fig.6 Pressure distributions on upstream surfaces of dams with a bottom inclination of 30°

Fig.7 Pressure distributions on upstream surfaces of dams with a bottom inclination of 45°

Fig.8 Pressure distributions on upstream surfaces of dams with a bottom inclination of 60°

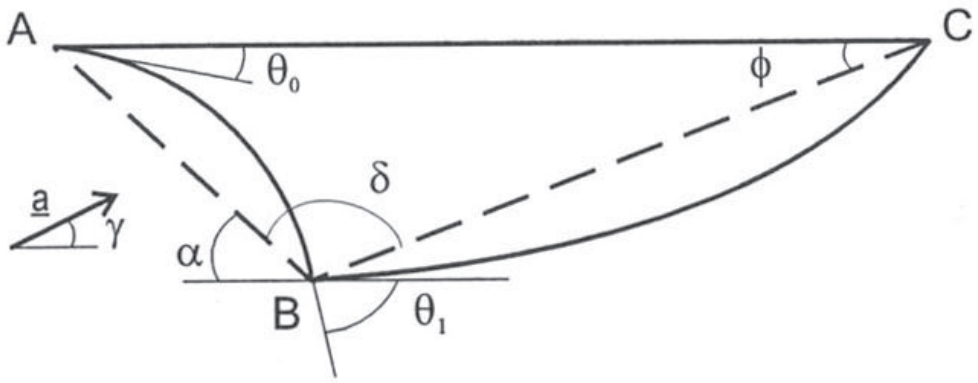

Fig. 1

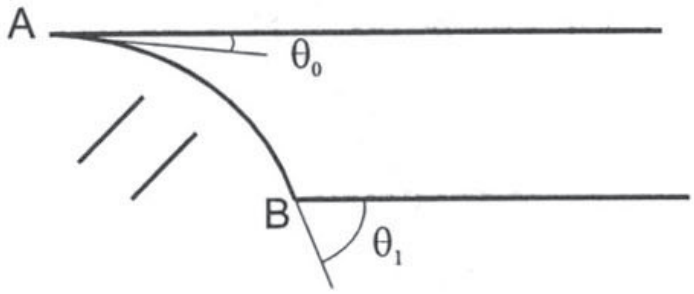

Fig. 2

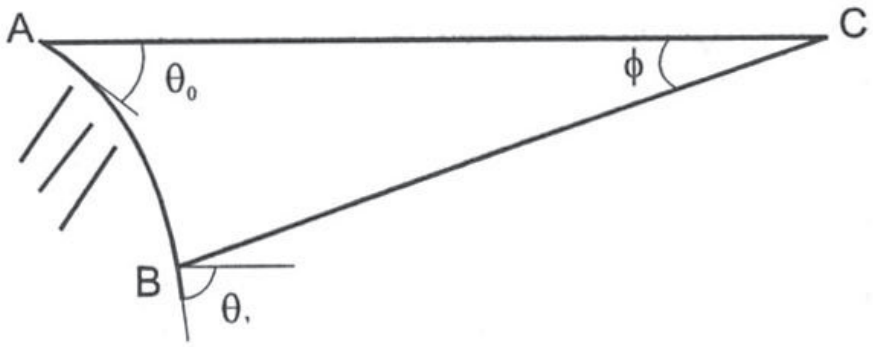

Fig. 3

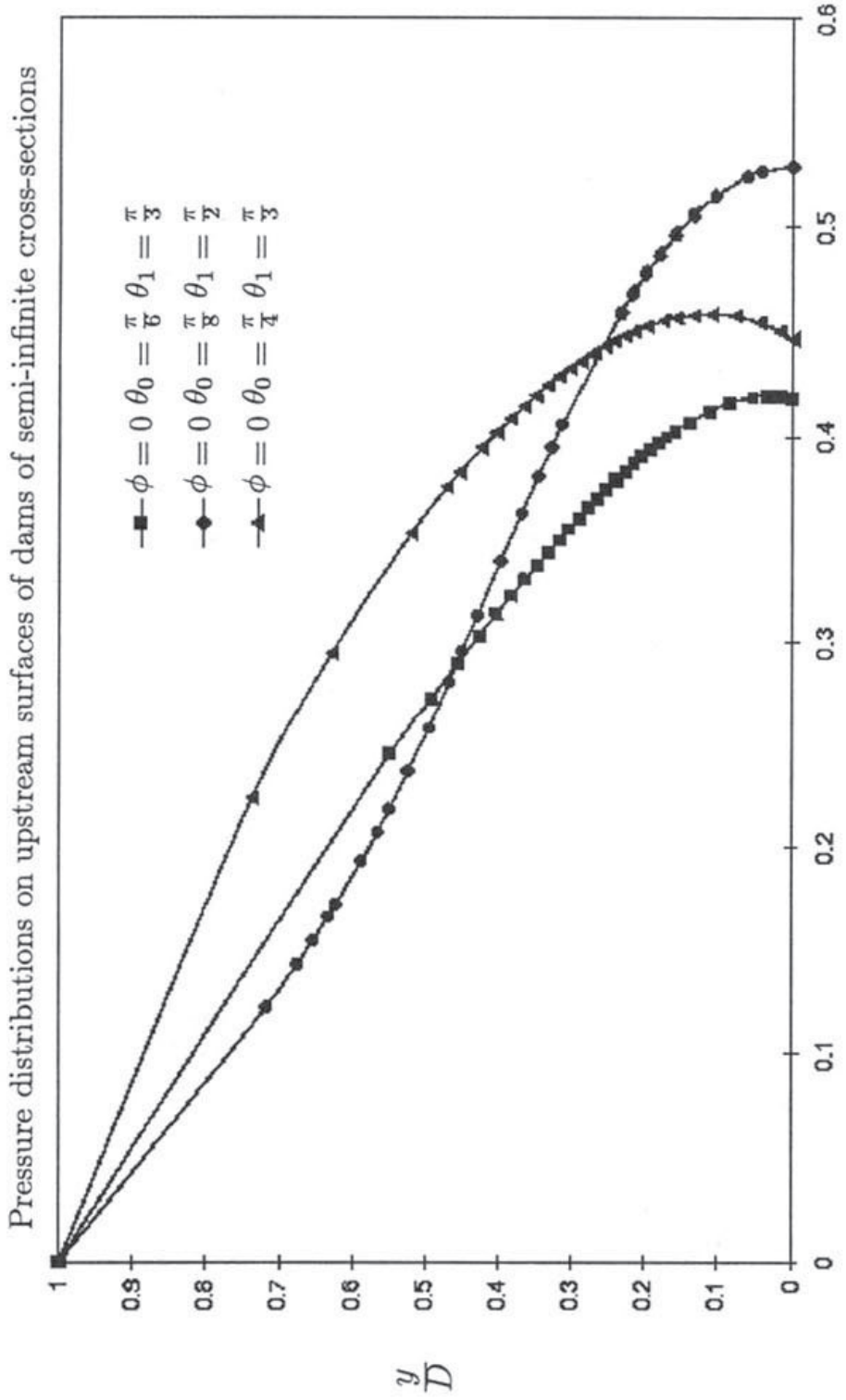
Pressure distributions on upstream surfaces of dams of semi-infinite cross-sections
$\phi = 0\ \theta_0 = \frac{\pi}{6}\ \theta_1 = \frac{\pi}{3}$
$\phi = 0\ \theta_0 = \frac{\pi}{8}\ \theta_1 = \frac{\pi}{2}$
$\phi = 0\ \theta_0 = \frac{\pi}{4}\ \theta_1 = \frac{\pi}{3}$
$\frac{y}{D}$
$\frac{P}{\rho a_o D}$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6

Fig. 4

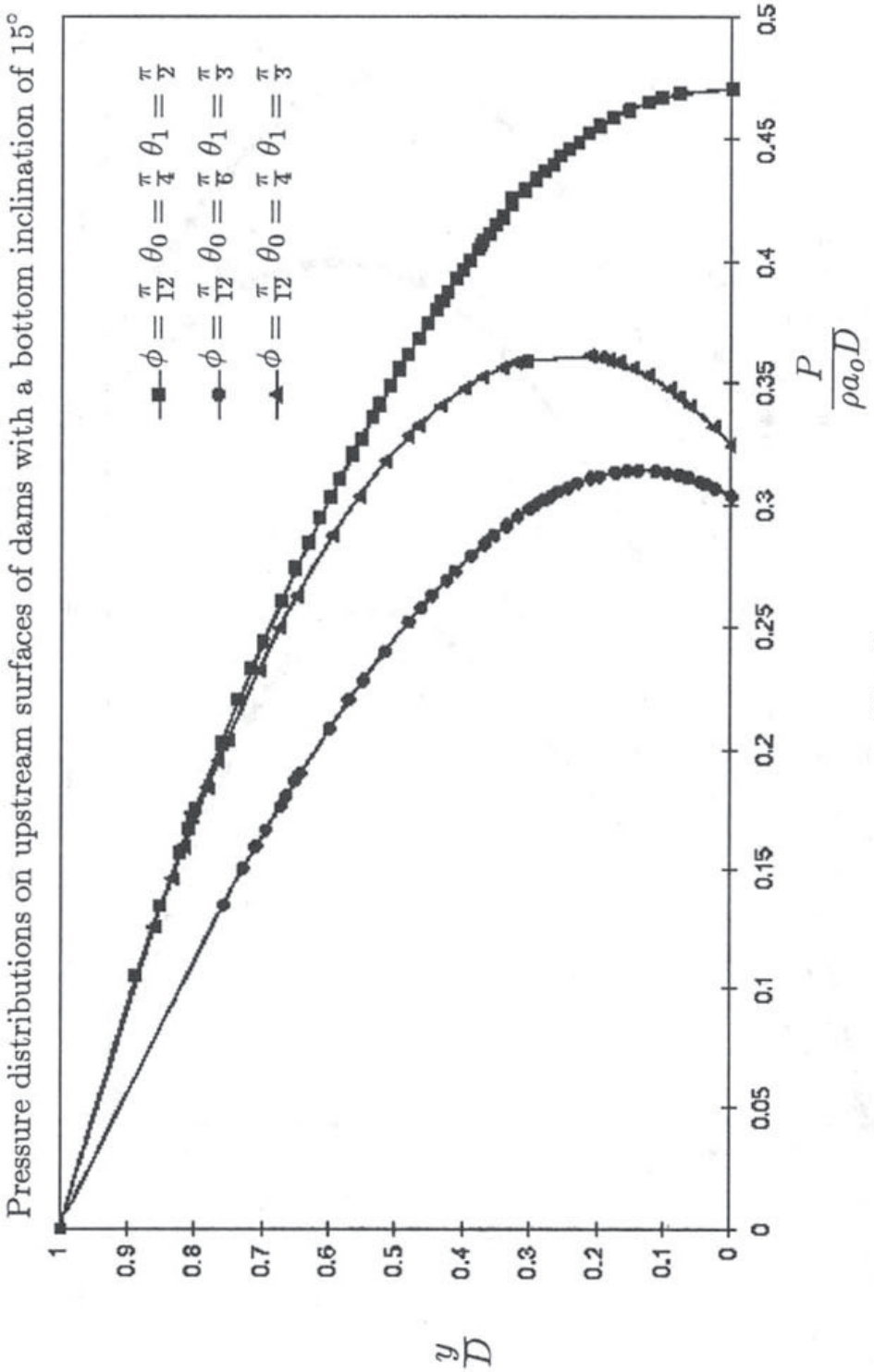
Pressure distributions on upstream surfaces of dams with a bottom inclination of 15°
$\phi = \frac{\pi}{12}$ $\theta_0 = \frac{\pi}{4}$ $\theta_1 = \frac{\pi}{2}$
$\phi = \frac{\pi}{12}$ $\theta_0 = \frac{\pi}{6}$ $\theta_1 = \frac{\pi}{3}$
$\phi = \frac{\pi}{12}$ $\theta_0 = \frac{\pi}{4}$ $\theta_1 = \frac{\pi}{3}$
$\frac{y}{D}$
$\frac{P}{\rho a_o D}$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.05
0.15
0.25
0.35
0.45

Fig. 5

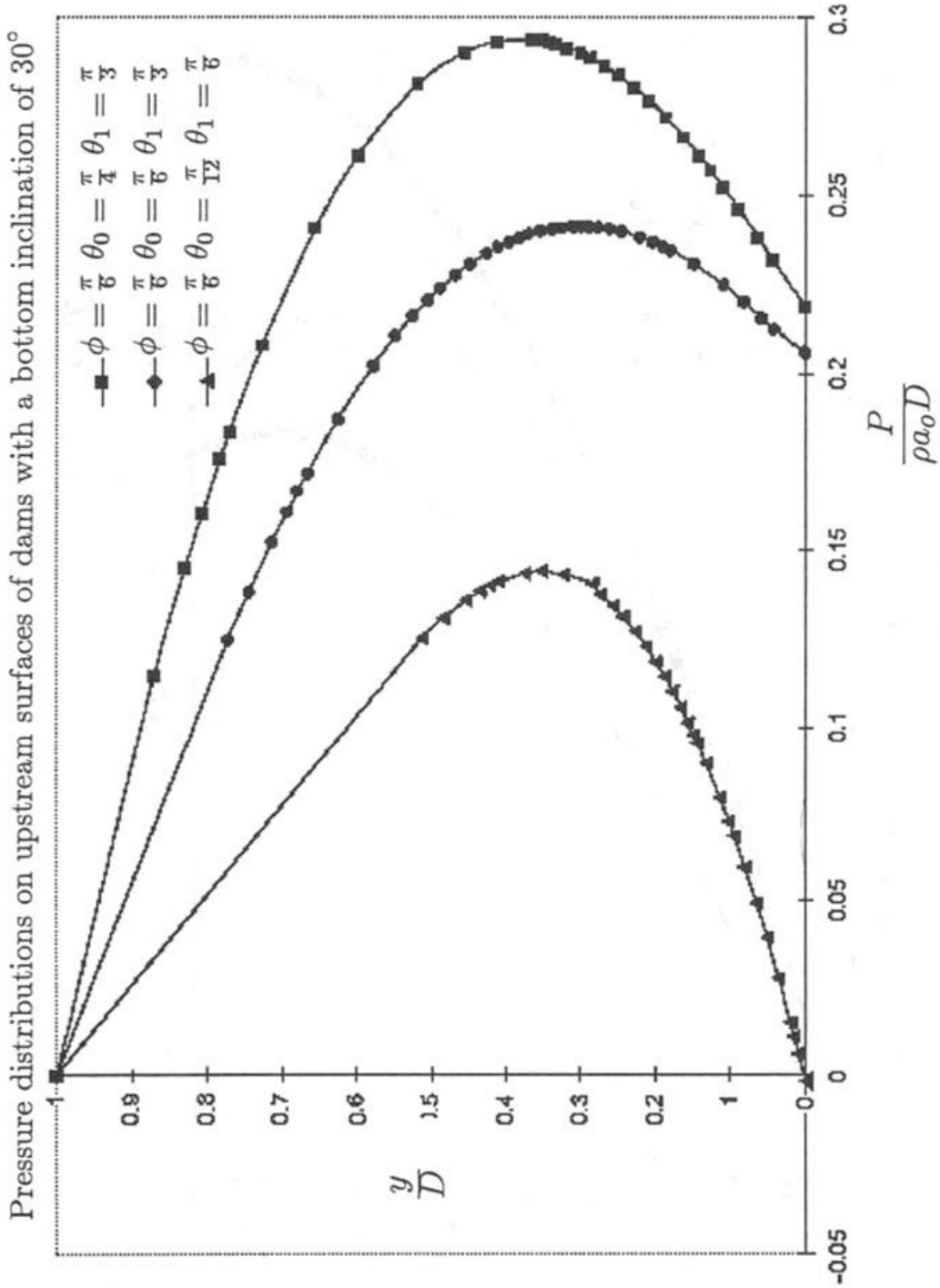
Pressure distributions on upstream surfaces of dams with a bottom inclination of 30°
$\phi = \frac{\pi}{6}$ $\theta_0 = \frac{\pi}{4}$ $\theta_1 = \frac{\pi}{3}$
$\phi = \frac{\pi}{6}$ $\theta_0 = \frac{\pi}{6}$ $\theta_1 = \frac{\pi}{3}$
$\phi = \frac{\pi}{6}$ $\theta_0 = \frac{\pi}{12}$ $\theta_1 = \frac{\pi}{6}$
$\frac{y}{D}$
$\frac{P}{\rho a_o D}$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.05
0.05
0.15
0.25
0.3

Fig. 6

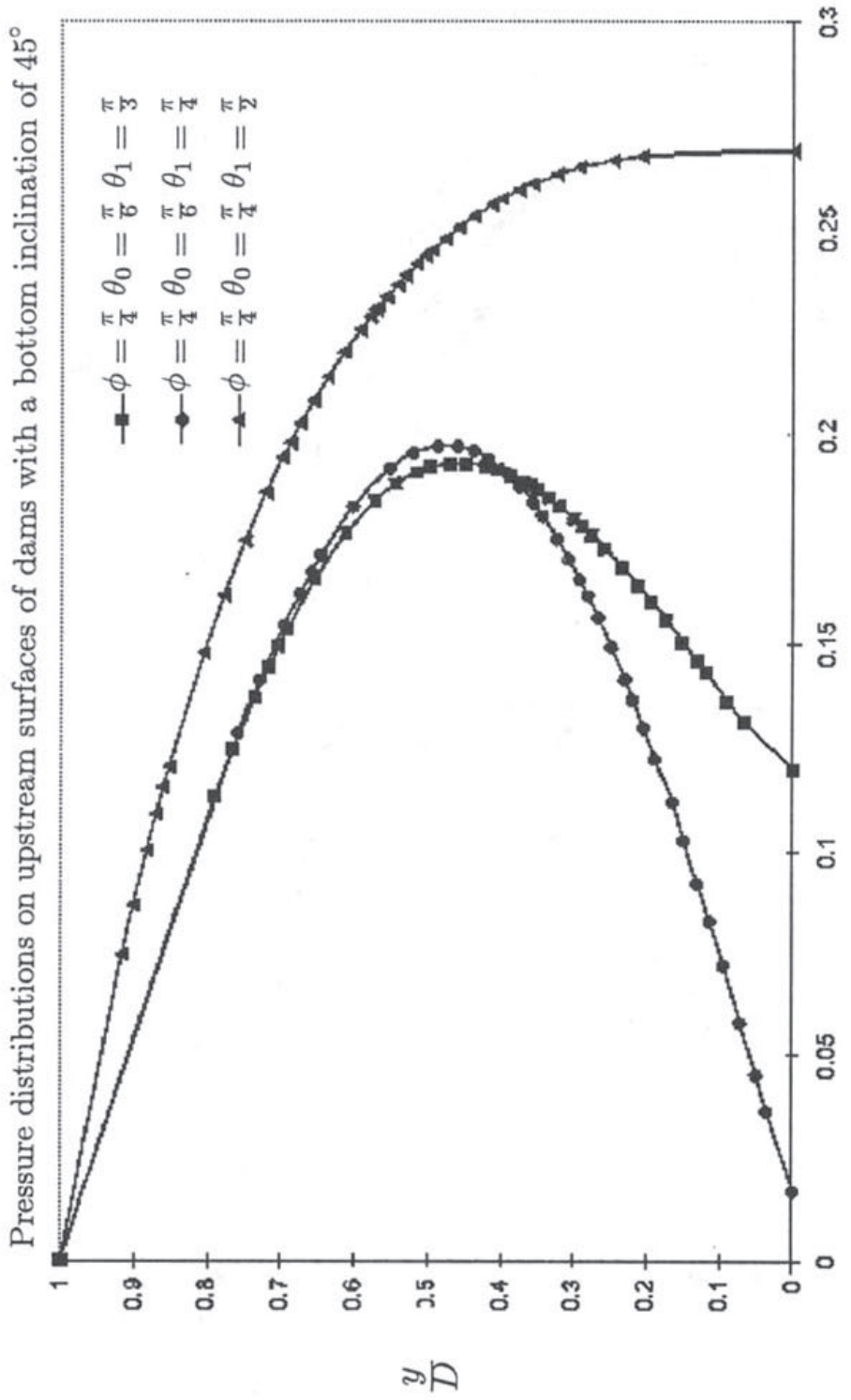
Pressure distributions on upstream surfaces of dams with a bottom inclination of 45°
$\phi = \frac{\pi}{4}\ \theta_0 = \frac{\pi}{6}\ \theta_1 = \frac{\pi}{3}$
$\phi = \frac{\pi}{4}\ \theta_0 = \frac{\pi}{6}\ \theta_1 = \frac{\pi}{4}$
$\phi = \frac{\pi}{4}\ \theta_0 = \frac{\pi}{4}\ \theta_1 = \frac{\pi}{2}$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$\frac{y}{D}$
0
0.05
0.1
0.15
0.2
0.25
0.3
$\frac{P}{\rho a_o D}$

Fig. 7

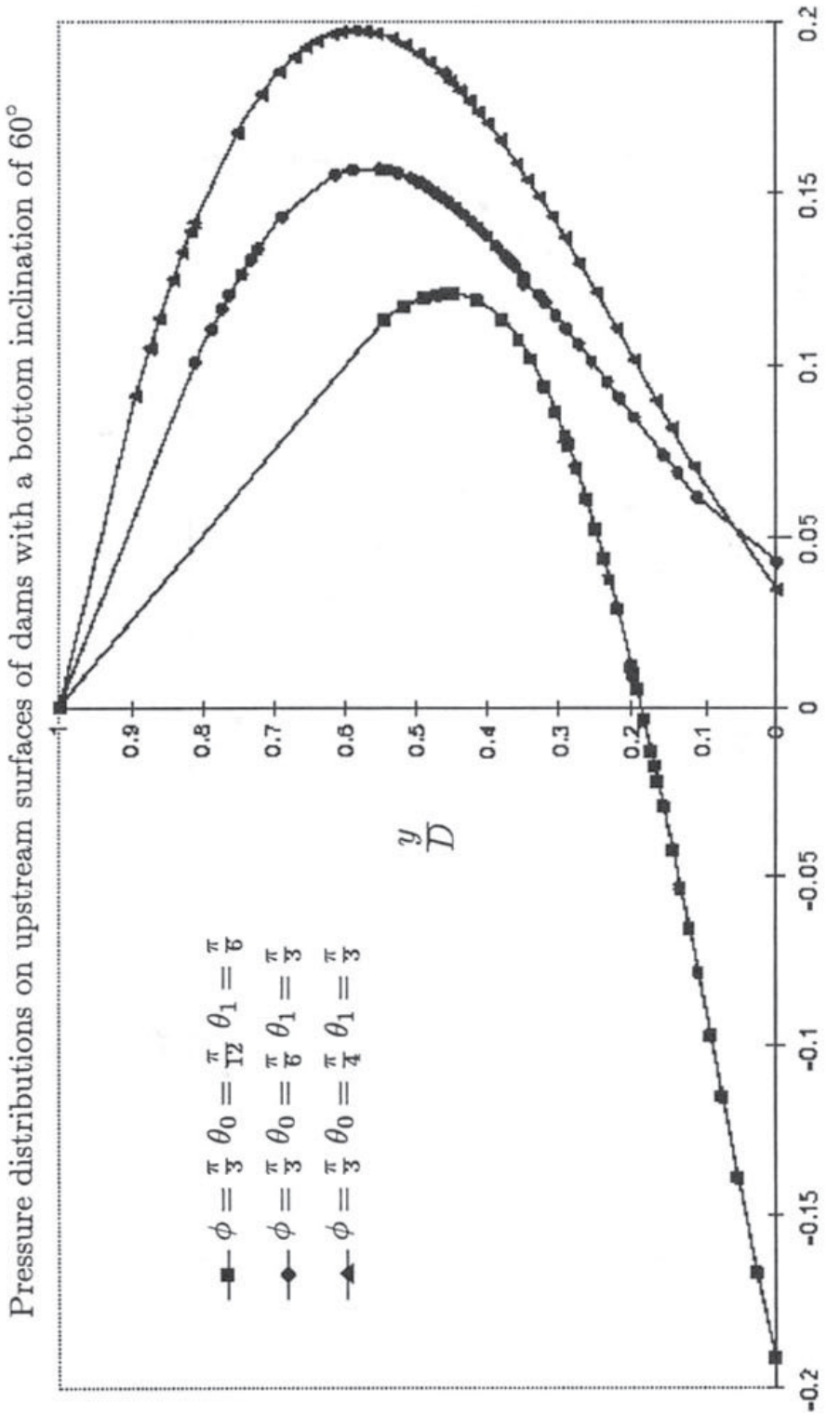
Pressure distributions on upstream surfaces of dams with a bottom inclination of 60°
$\phi = \frac{\pi}{3}$ $\theta_0 = \frac{\pi}{12}$ $\theta_1 = \frac{\pi}{6}$
$\phi = \frac{\pi}{3}$ $\theta_0 = \frac{\pi}{6}$ $\theta_1 = \frac{\pi}{3}$
$\phi = \frac{\pi}{3}$ $\theta_0 = \frac{\pi}{4}$ $\theta_1 = \frac{\pi}{3}$
$\frac{y}{D}$
$\frac{P}{\rho a_o D}$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2

Fig. 8